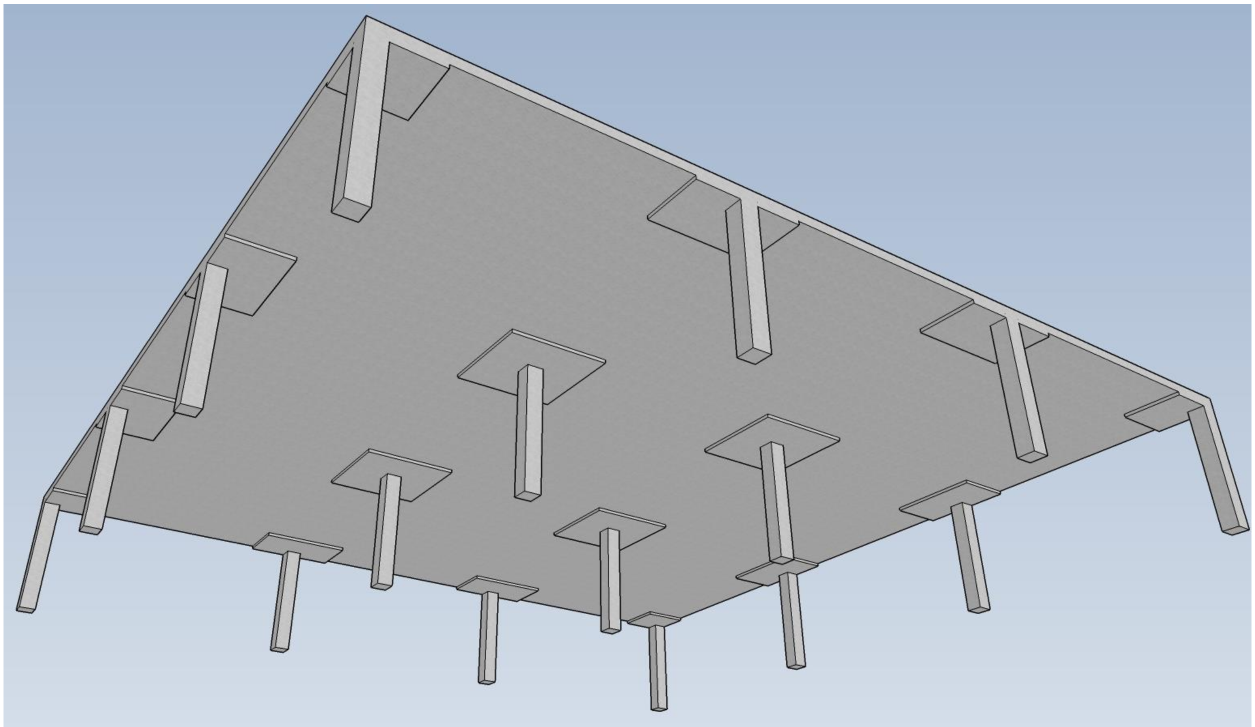
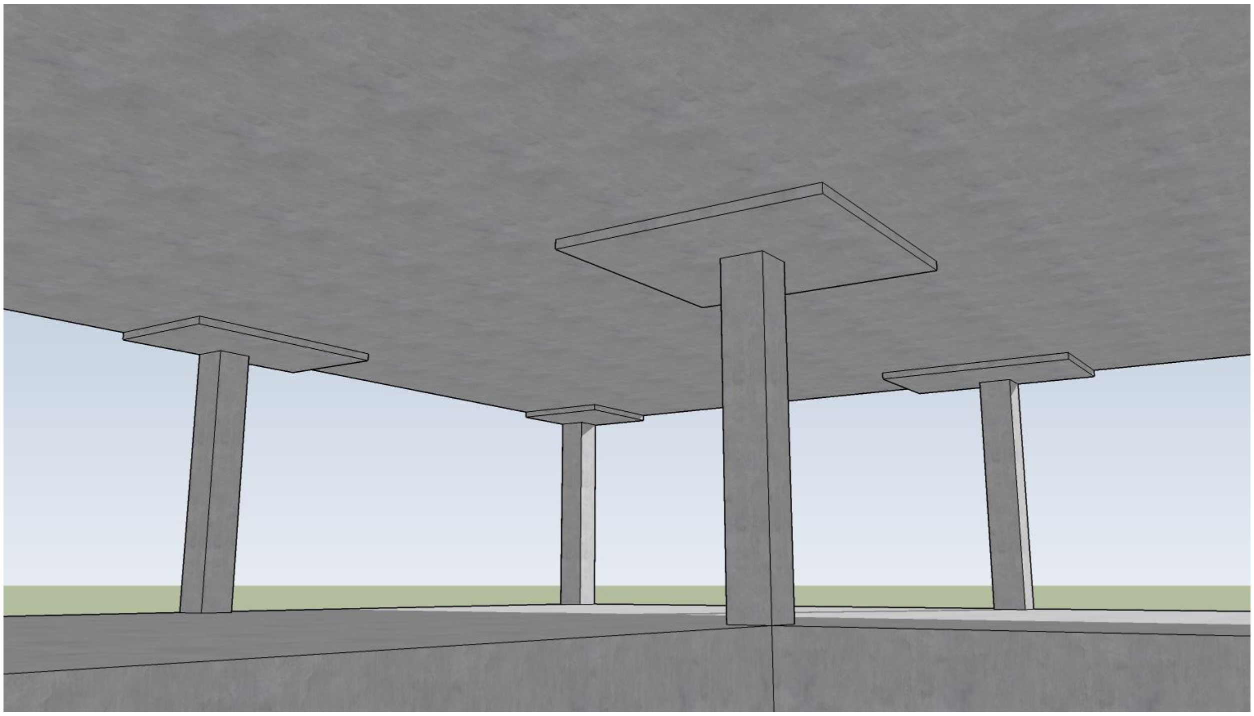
Two-Way Flat Slab (Concrete Floor with Drop Panels) System Analysis and Design (CSA A23.3-14)





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Design the concrete floor slab system shown below for an intermediate floor considering partition weight = 1 kN/m^2, and unfactored live load = 3 kN/m^2. The lateral loads are independently resisted by shear walls. The use of flat plate system will be checked. If the use of flat plate is not adequate, the use of flat slab system with drop panels will be investigated. Flat slab concrete floor system is similar to the flat plate system. The only exception is that the flat slab uses drop panels (thickened portions around the columns) to increase the nominal shear strength of the concrete at the critical section around the columns. The Elastic Frame Method (EFM) shown in CSA A23.3-14 is used in this example. The hand solution from EFM is also used for a detailed comparison with the model results of spSlab engineering software program.

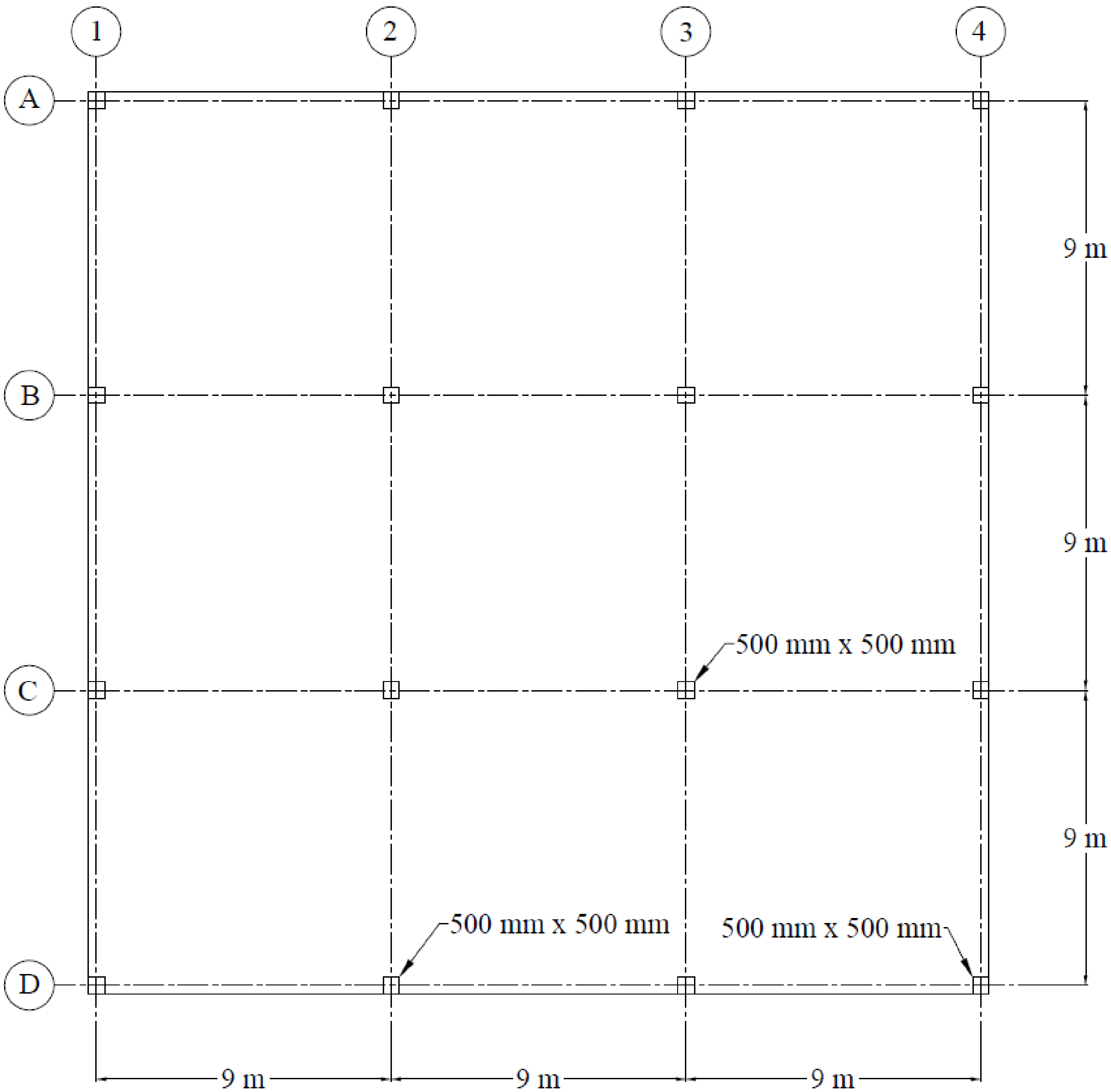


Figure 1 - Two-Way Flat Concrete Floor System

Contents

Code

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 "Design of Concrete Structures"

Reference

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Design Data

Story Height = 4 m (provided by architectural drawings)

Superimposed Dead Load, *SDL* =1 kN/m^2

Live Load, LL = 3 kN/m^2

f_c' = 35 MPa (for slab)

f_c' = 42 MPa (for columns)

f_y = 400 MPa

Solution

1. Preliminary member sizing

<u>For slabs without Drop Panels</u>

a. <u>Slab minimum thickness – Deflection</u> ***<u>CSA A23.3-14 (13.2.3)</u>***

In lieu of detailed calculation for deflections, CSA A23.3 Code gives minimum slab thickness for two-way construction without interior beams in ***Clause 13.2.3***.

For this flat plate slab systems the minimum slab thicknesses per **CSA A23.3-14** are:

Exterior Panels: $h_{s,\min} = 1.1 \times \dfrac{l_n\left(0.6 + f_y / 1,000\right)}{30} = 311.7 \text{ mm}$ ***<u>CSA A23.3-14 (13.2.3)</u>***

But not less than 120 mm. ***<u>CSA A23.3-14 (13.2.1)</u>***

Exterior Panels: $h_{s,\min} = \dfrac{l_n\left(0.6 + f_y / 1,000\right)}{30} = 283.3 \text{ mm}$ ***<u>CSA A23.3-14 (13.2.3)</u>***

But not less than 120 mm. ***<u>CSA A23.3-14 (13.2.1)</u>***

Where l_n = length of clear span in the long direction = 9,000 – 500 = 8,500 mm

Try 300 mm. slab for all panels (self-weight = 24 kN/m^3 × 0.3 m = 7.2 kN/m^2)

b. <u>Slab shear strength – one way shear</u>

At a preliminary check level, the use of average effective depth would be sufficient. However, after determining the final depth of the slab, the exact effective depth will be used in flexural, shear and deflection calculations. Evaluate the average effective depth (Figure 2):

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 300 - 20 - 16 - \frac{16}{2} = 256 \text{ mm}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 300 - 20 - \frac{16}{2} = 272 \text{ mm}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{256 + 272}{2} = 264 \text{ mm}$$

Where:

c_{clear} = 20 mm **_CSA A23.3-14 (Annex A. Table 17)_**

d_b = 16 mm for 15M steel bar

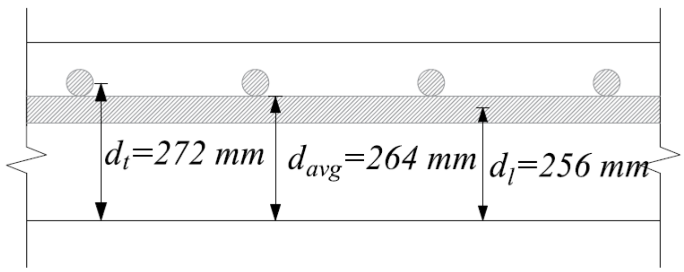


Figure 2 - Two-Way Flat Concrete Floor System

Factored dead load, $w_{df} = 1.25 \times (7.2 + 1) = 10.25 \text{ kN/m}^2$ **_CSA A23.3-14 (Annex C. Table C.1 a)_**

Factored live load, $w_{lf} = 1.5 \times 3 = 4.5 \text{ kN/m}^2$

Total factored load, $w_f = 14.75 \text{ kN/m}^2$

Check the adequacy of slab thickness for beam action (one-way shear) **_CSA A23.3-14 (13.3.6)_**

At an interior column:

The critical section for one-way shear is extending in a plane across the entire width and located at a distance, d_v from the face of support or concentrated load (see Figure 3). **_CSA A23.3-14 (13.3.6.1)_**
Consider a 1 m wide strip

Tributary are for one-way shear is $A_{Tributary} = \left(\frac{\left[\left(\frac{9,000}{2} \right) - \left(\frac{500}{2} \right) - 264 \right] \times (1,000)}{1,000^2} \right) = 3.986 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 14.75 \times 3.986 = 58.79 \text{ kN}$$

$V_c = \varphi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ **_CSA A23.3-14 (Eq.11.6)_**

Where:

$\lambda = 1$ For normal weight concrete **_CSA A23.3-14 (8.6.5)_**

$\beta = 0.21$ For slabs with overall thickness not greater than 350 mm **_CSA A23.3-14 (11.3.6.2)_**

$d_v = Max\ (0.9d,\ 0.75h) = 237.6$ mm **_CSA A23.3-14 (3.2)_**

$\sqrt{f'_c} = 5.29$ MPa < 8 MPa **_CSA A23.3-14 (11.3.4)_**

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{35} \times 1,000 \times \frac{237.6}{1,000} = 191.87 \text{ kN } > V_f$$

Slab thickness of 300 mm is adequate for one-way shear.

c. Slab shear strength – two-way shear

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

Tributary area for two-way shear is $A_{Tributary} = (9 \times 9) - \left(\frac{400+264}{1,000}\right)^2 = 80.42 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 80.42 \times 14.75 = 1,186.1 \text{ kN}$$

$$v_f = \frac{V_f}{b_o d} = \frac{1,186.1}{3,056 \times 264} = 1.47 \text{ MPa}$$

$v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c}$ **_CSA A23.3-14 (13.3.4.1)_**

$$v_c = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa} > v_f$$

Slab thickness of 300 mm is not adequate for two-way shear. It is good to mention that the factored shear (V_f) used in the preliminary check does not include the effect of the unbalanced moment at supports. Including this effect will lead to an increase of V_f value as shown later in section 4.2.

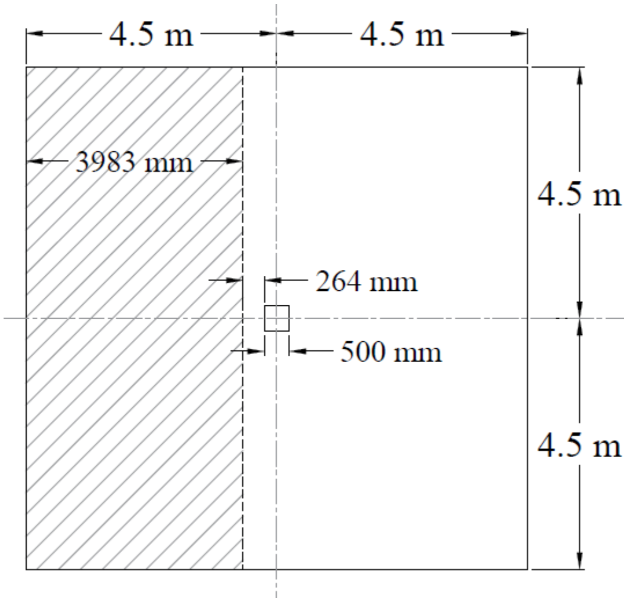


Figure 3 – Critical Section for One-Way Shear

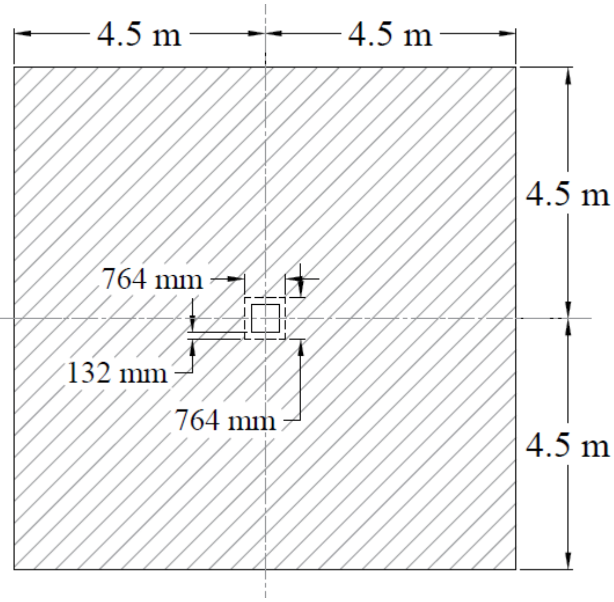


Figure 4 – Critical Section for Two-Way Shear

In this case, four options could be used: 1) to increase the slab thickness, 2) to increase columns cross sectional dimensions or cut the spacing between columns (reducing span lengths), however, this option is assumed to be not permissible in this example due to architectural limitations, 3) to use headed shear reinforcement, or 4) to use drop panels. In this example, the latter option will be used to achieve better understanding for the design of two-way slab with drop panels often called flat slab.

Check the drop panel dimensional limitations as follows:

1) The additional thickness of the drop panel below the soffit of the slab (Δ_h) shall not be taken larger than h_s.

CSA A23.3-14 (13.2.4)

Since the slab thickness (h_s) is 260 mm (see page 7), the thickness of the drop panel should be at less than 260 mm.

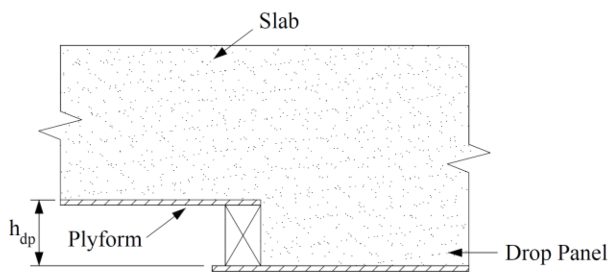
Drop panel dimensions are also controlled by formwork considerations. The following Figure shows the standard lumber dimensions that are used when forming drop panels. Using other depths will unnecessarily increase formwork costs. The Δ_h dimension will be taken as the lumber dimension plus the thickness of one sheet of plywood (19 mm).

For nominal lumber size:

h_{dp} = 38+19 = 57 mm or h_{dp} = 89+19 = 108 mm

Try h_{dp} = 108 mm < 260 mm

The total thickness including the slab and the drop panel (h) = h_s + h_{dp} = 260 + 108 = 368 mm



Nominal Lumber Size, mm	Actual Lumber Size, mm	Plyform Thickness, mm	h_{dp}, mm
2x	38	19	57
4x	89	19	108

Figure 5 – Drop Panel Formwork Details

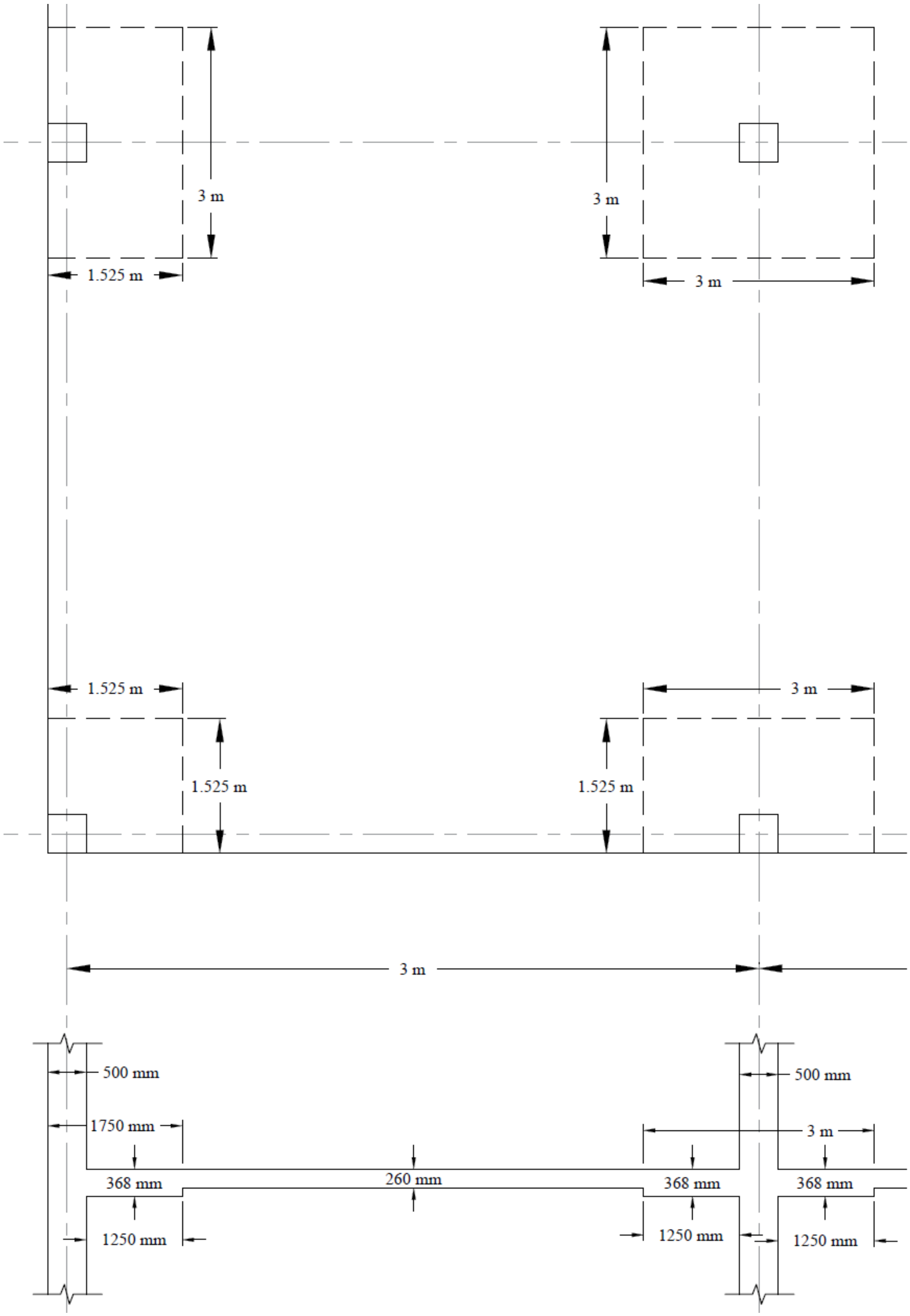


Figure 6 – Drop Panels Dimensions

For Flat Slab (with Drop Panels)

For slabs with changes in thickness and subjected to bending in two directions, it is necessary to check shear at multiple sections as defined in the ***CSA A23.3-14***. The critical sections for two-way action shall be located with respect to:

1) Perimeter of the concentrated load or reaction area. ***CSA A.23.3-14 (13.3.3.1)***

2) Changes in slab thickness, such as edges of drop panels. ***CSA A.23.3-14 (13.3.3.2)***

a. Slab minimum thickness – Deflection

In lieu of detailed calculation for deflections, CSA A23.3 Code gives minimum slab thickness for two-way construction with drop panel in ***Clause 13.2.4.***

For this flat plate slab systems the minimum slab thicknesses per **CSA A23.3-14** are:

The value of $2x_d/l_n$ is not known at this point. The upper limit is 1/4. A reasonable preliminary estimate is 1/6.

Exterior Panels: $h_{s,\min} = 1.1 \times \left(\dfrac{l_n \left(0.6 + f_y / 1{,}000\right)}{30} - \dfrac{2x_d}{l_n} \right) = 272.1 \text{ mm}$ ***CSA A23.3-14 (13.2.4)***

But not less than 120 mm. ***CSA A23.3-14 (13.2.1)***

Interior Panels: $h_{s,\min} = \dfrac{l_n \left(0.6 + f_y / 1{,}000\right)}{30} - \dfrac{2x_d}{l_n} = 247.3 \text{ mm}$ ***CSA A23.3-14 (13.2.4)***

But not less than 120 mm. ***CSA A23.3-14 (13.2.1)***

Where

l_n = length of clear span in the long direction = 9,000 – 500 = 8,500 mm

Try 260 mm slab for all panels

Self-weight for slab section without drop panel = 24 kN/m^3 × 0.26 m = 6.24 kN/m^2

Self-weight for slab section with drop panel = 24 kN/m^3 × 0.368 m = 8.83 kN/m^2

b. Slab shear strength – one way shear

For critical section at distance *d* from the edge of the column (slab section with drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 368 - 20 - 16 - \frac{16}{2} = 324 \text{ mm}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 368 - 20 - \frac{16}{2} = 340 \text{ mm}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{324 + 340}{2} = 332 \text{ mm}$$

Where:

c_{clear} = 20 mm **CSA A23.3-14 (Annex A. Table 17)**

d_b = 16 mm for 15M steel bar

Factored dead load $\rightarrow$ $w_{df} = 1.25 \times (8.83 + 1) = 12.29 \text{ kN/m}^2$ **CSA A23.3-14 (Annex C. Table C.1 a)**

Factored live load $\rightarrow$ $w_{lf} = 1.5 \times 3 = 4.5 \text{ kN/m}^2$

Total factored load $\rightarrow$ $w_f = 12.29 + 4.5 = 16.79 \text{ kN/m}^2$

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior column

CSA A23.3-14 (13.3.6)

Consider a 1 m wide strip. The critical section for one-way shear is located at a distance d_v, from the edge of the column (see Figure 7)

Tributary area for one-way shear is $A_{Tributary} = \left(\dfrac{\left[\left(\dfrac{9{,}000}{2} \right) - \left(\dfrac{500}{2} \right) - 299 \right] \times (1{,}000)}{1{,}000^2} \right) = 3.95 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 16.79 \times 3.95 = 66.34 \text{ kN}$$

$V_c = \varphi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ **CSA A23.3-14 (Eq. 11.6)**

Where $\lambda = 1$ for normal weight concrete

This slab contains no transverse reinforcement and it is assumed the specified nominal maximum size of coarse aggregate is not less than 20 mm, β shall be taken as: **CSA A23.3-14 (11.3.6.3)**

$$\beta = \frac{230}{(1{,}000 + d_v)} = \frac{230}{(1{,}000 + 331.2)} = 0.173$$

$d_v = Max\ (0.9d,\ 0.75h) = 299 \text{ mm}$ **CSA A23.3-14 (3.2)**

$$V_c = 0.65 \times 1 \times 0.17 \times \sqrt{35} \times 1{,}000 \times \frac{299}{1{,}000} = 198.52 \text{ kN} > V_u$$

Slab thickness of 368 mm is adequate for one-way shear for the first critical section (from the edge of the column).

For critical section at the edge of the drop panel (slab section without drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 260 - 20 - 16 - \frac{16}{2} = 216 \text{ mm}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 260 - 20 - \frac{16}{2} = 232 \text{ mm}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{216 + 232}{2} = 224 \text{ mm}$$

Where:

c_{clear} = 20 mm **CSA A23.3-14 (Annex A. Table 17)**

d_b = 16 mm for 15M steel bar

Factored dead load $\rightarrow w_{df} = 1.25 \times (6.24 + 1) = 9.05 \text{ kN/m}^2$ **CSA A23.3-14 (Annex C. Table C.1 a)**

Factored live load $\rightarrow w_{lf} = 1.5 \times 3 = 4.5 \text{ kN/m}^2$

Total factored load $\rightarrow w_f = 9.05 + 4.5 = 13.55 \text{ kN/m}^2$

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior drop panel. **CSA A23.3-14 (13.3.6)**

Consider a 1 m wide strip. The critical section for one-way shear is located at a distance, d_v from the face of support (see Figure 7)

Tributary area for one-way shear is $A_{Tributary} = \left(\frac{\left[\left(\frac{9,000}{2} \right) - \left(\frac{3,000}{2} \right) - 202 \right] \times (1,000)}{1,000^2} \right) = 2.80 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 13.55 \times 2.8 = 37.92 \text{ kN}$$

$V_c = \varphi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ **CSA A23.3-14 (Eq. 11.6)**

Where $\lambda = 1$ for normal weight concrete

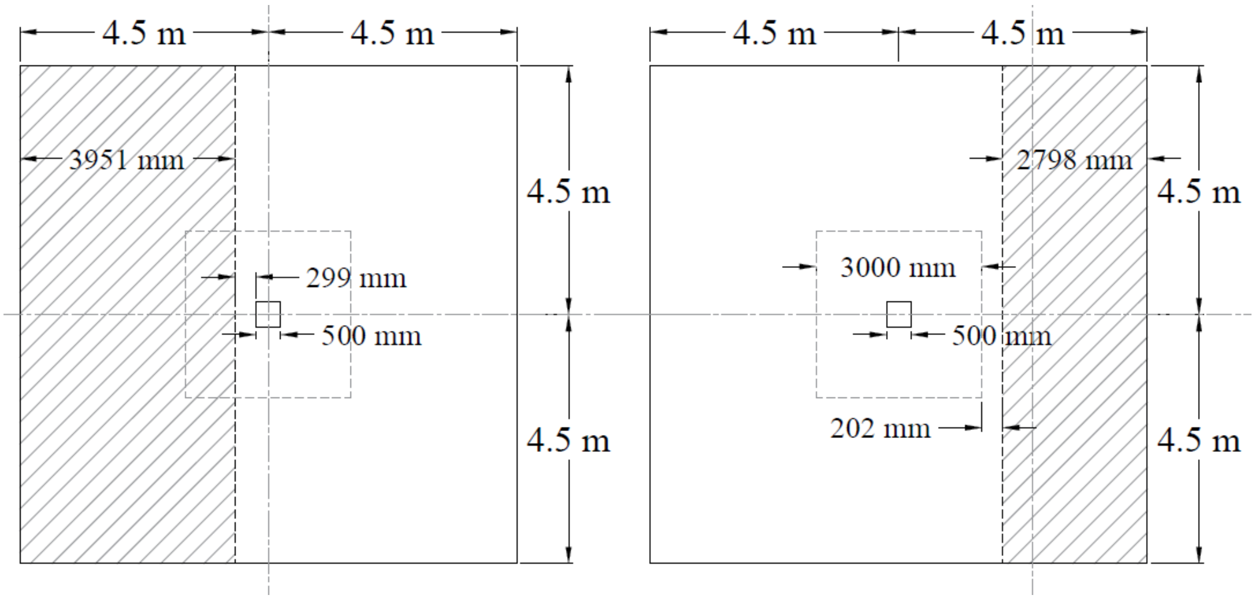
$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm **CSA A23.3-14 (11.3.6.2)**

$d_v = Max\ (0.9d,\ 0.75h) = 202 \text{ mm}$ **CSA A23.3-14 (3.2)**

$\sqrt{f'_c} = 5.29 \text{ MPa} < 8 \text{ MPa}$ **CSA A23.3-14 (11.3.4)**

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{35} \times 1,000 \times \frac{202}{1,000} = 180.9 \text{ kN} > V_f$$

Slab thickness of 260 mm is adequate for one-way shear for the second critical section (from the edge of the drop panel).



Critical Section from the Edge of the Column

Critical Section from the Edge of the Drop Panel

Figure 7 – Critical Sections for One-Way Shear

c. Slab shear strength – two-way shear

For critical section at distance *d*/2 from the edge of the column (slab section with drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 8):

Tributary area for two-way shear is $A_{Tributary} = (9\times 9) - (0.5 + 0.224)^2 = 80.31 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 16.79 \times 80.31 = 1{,}348.37 \text{ kN}$$

$$v_f = \frac{V_f}{b_o d} = \frac{1{,}348.4}{3{,}328 \times 332} = 1.22 \text{ MPa}$$

$$v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c}$$ **CSA A23.3-14 (13.3.4.1)**

$$v_c = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa} > v_f$$

Tributary area for two-way shear is $A_{Tributary} = (9\times 9) - (3 + 0.224)^2 = 70.61 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 13.55 \times 70.61 = 956.7 \text{ kN}$$

$$v_f = \frac{V_f}{b_o d} = \frac{956.7}{12{,}896 \times 224} = 0.33 \text{ MPa}$$

$$v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c}$$ **CSA A23.3-14 (13.3.4.1)**

$$v_c = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa} > v_f$$

Slab thickness of 260 mm is adequate for two-way shear for the first critical section (from the edge of the column).

For critical section at the edge of the drop panel (slab section without drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior drop panel (Figure 8):

Tributary area for two-way shear is $A_{Tributary} = (9\times 9) - (3+0.224)^2 = 70.61 \text{ m}^2$

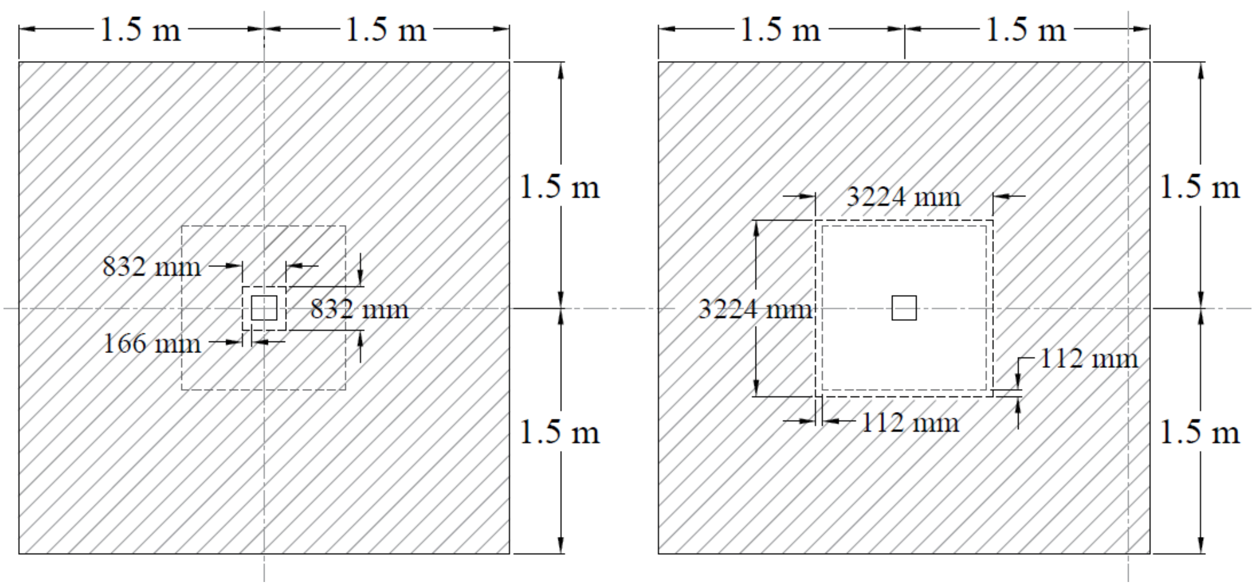
$$V_f = w_f \times A_{Tributary} = 13.55\times 70.61 = 956.7 \text{ kN}$$

$$v_f = \frac{V_f}{b_o d} = \frac{956.7}{12,896\times 224} = 0.33 \text{ MPa}$$

$$v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c}$$ **CSA A23.3-14 (13.3.4.1)**

$$v_c = 0.38\times 1\times 0.65\times\sqrt{35} = 1.46 \text{ MPa} > v_f$$

Slab thickness of 260 mm is adequate for two-way shear for the second critical section (from the edge of the drop panel).



Critical Section from the Edge of the Column   Critical Section from the Edge of the Drop Panel

Figure 8 – Critical Sections for Two-Way Shear

d. Column dimensions - axial load

Check the adequacy of column dimensions for axial load:

Tributary area for interior column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = 9 \times 9 = 81 \text{ m}^2$$

Tributary area for interior column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = 3 \times 3 = 9 \text{ m}^2$$

Assuming five story building

$$P_f = n \times w_f \times A_{Tributary} = 5 \times (13.55 \times 81 + 3.24 \times 9) = 5,633.5 \text{ kN}$$

Assume 500 mm square column with 12 – 30M vertical bars with design axial strength, $P_{r,max}$ of

$$P_{r,\max} = (0.2 + 0.002h)P_{ro} \le 0.80P_{ro} \text{ (For tied column along full length)}$$ ***CSA A23.3-14 (Eq. 10.9)***

$$P_{ro} = \alpha_1 \phi_c f'_c (A_g - A_{st} - A_t - A_p) + \phi_s \text{ f}_y \text{ A}_{st} + \phi_\alpha F_y A_t - f_{pr} A_p$$ ***CSA A23.3-14 (Eq. 10.11)***

$$P_{ro} = 0.8 \times 0.65 \times 35 \times (500 \times 500 - 8 \times 700) + 0.85 \times 400 \times (12 \times 700) + 0 = 7,239.4 \text{ kN}$$

$$P_{r,\max} = (0.2 + 0.002 \times 500) \times 7,239.4 \le 0.80 \times 7,239.4$$

$$P_{r,\max} = 5,791.5 \text{ kN} < P_f$$

Where:

$$\alpha_1 = 0.85 - 0.0015 f'_c = 0.85 - 0.0015 \times 35 = 0.8 > 0.67$$ ***CSA A23.3-14 (Eq. 10.1)***

Column dimensions of 500 mm × 500 mm are adequate for axial load.

2. Flexural Analysis and Design

CSA A23.3 states that a slab system shall be designed by any procedure satisfying equilibrium and geometric compatibility, provided that strength and serviceability criteria are satisfied. Distinction of two-systems from one-way systems is given by ***CSA A23.3-14 (3.2.2)***

CSA A23.3-14 permits the use of Direct Design Method (DDM) and Elastic Frame Method (EFM) for the gravity load analysis of orthogonal frames and is applicable to flat plates, flat slabs, and slabs with beams. The following sections outline the solution per EFM and spSlab software. For the solution per DDM, check the flat plate example.

2.1. Elastic Frame Method (EFM)

EFM (as known as Equivalent Frame Method in the ACI 318) is the most comprehensive and detailed procedure provided by the CSA A23.3 for the analysis and design of two-way slab systems where these systems may, for purposes of analysis, be considered a series of plane frames acting longitudinally and transversely through the building. Each frame shall be composed of equivalent line members intersecting at member centerlines, shall follow a column line, and shall include the portion of slab bounded laterally by the centerline of the panel on each side. ***CSA A23.3-14 (13.8.1.1)***

Probably the most frequently used method to determine design moments in regular two-way slab systems is to consider the slab as a series of two-dimensional frames that are analyzed elastically. When using this analogy, it is essential that stiffness properties of the elements of the frame be selected to properly represent the behavior of the three-dimensional slab system.

2.1.1. Limitations for use of elastic frame method

In EFM, live load shall be arranged in accordance with 13.8.4 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***CSA A23.3-14 (13.8.4)***

Complete analysis must include representative interior and exterior equivalent elastic frames in both the longitudinal and transverse directions of the floor. ***CSA A23.3-14 (13.8.1.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***CSA A23.3-14 (13.2.2)***

For slab systems with beams between supports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 2. ***CSA A23.3-14 (13.2.2)***

Column offsets are not greater than 20% of the span (in the direction of offset) from either axis between centerlines of successive columns. ***CSA A23.3-14 (13.2.2)***

The reinforcement is placed in an orthogonal grid. ***CSA A23.3-14 (13.2.2)***

2.1.2. Frame members of elastic frame

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors *k*, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a. Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{\ell_1} = \frac{500}{9,000} = 0.056 \ , \ \frac{c_{N2}}{\ell_2} = \frac{500}{9,000} = 0.056$$

For $c_{F1} = c_{N1}$, stiffness factors, $k_{NF} = k_{FN} = 5.55$ ***PCA Notes on ACI 318-11 (Table A2&3)***

Thus, $K_{sb} = k_{NF}\dfrac{E_{cs}I_s}{\ell_1} = 5.55\dfrac{E_{cs}I_s}{\ell_1}$ ***PCA Notes on ACI 318-11 (Table A2&3)***

$$K_{sb} = 5.55 \times 29{,}002 \times \frac{1.32 \times 10^{10}}{9{,}000} = 2.36 \times 10^{8} \text{ N.m}$$

Where, $I_s = \dfrac{\ell_s h^3}{12} = \dfrac{9{,}000 \times (260)^3}{12} = 1.32 \times 10^{10} \text{ mm}^4$

$$E_{cs} = (3{,}300\sqrt{f_c^{'}} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_{cs} = (3{,}300\sqrt{35} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 29{,}002 \text{ MPa}$$

Carry-over factor COF = 0.576 ***PCA Notes on ACI 318-11 (Table A2&3)***

Fixed-end moment, $FEM = \Sigma_{i=1}^{n} m_{NFi} \times w_i \times l_1^2$ ***PCA Notes on ACI 318-11 (Table A2&3)***

Uniform load fixed end moment coefficient, m_{NF1} = 0.0913

Fixed end moment coefficient for (b-a) = 0.2 when a = 0, m_{NF2} = 0.0163

Fixed end moment coefficient for (b-a) = 0.2 when a = 0.8, m_{NF3} = 0.0163

The coefficient of fixed end moment for (b-a) = 0.2 when a = 0.8 is taken as 0.0163 in order to be conservative and take the upper bound and be conservative.

b. Flexural stiffness of column members at both ends, K_c.

Referring to ***Table A7, Appendix 20A***,

For the Bottom Column (Below):

$t_a = 260/2 + 108 = 238$ mm , $t_b = 260/2 = 130$ mm

$$\frac{t_a}{t_b} = \frac{238}{130} = 1.83$$

$H = 4 \text{ m} = 4{,}000 \text{ mm}$, $H_c = 4{,}000 \text{ mm} - 260 \text{ mm} - 108 \text{ mm} = 3{,}632 \text{ mm}$

$$\frac{H}{H_c} = \frac{4{,}000}{3{,}632} = 1.10$$

Thus, $k_{AB} = 5.31$ and $C_{AB} = 0.55$ by interpolation.

$$K_{c,bottom} = \frac{5.31 E_{cc} I_c}{\ell_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,bottom} = 5.31 \times 31{,}047 \times \frac{5.21 \times 10^9}{4{,}000 \times 1{,}000} = 2.15 \times 10^8 \text{ N.m}$$

Where $I_c = \dfrac{c^4}{12} = \dfrac{(500)^4}{12} = 5.21 \times 10^9 \text{ mm}^4$

$$E_{cc} = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_{cc} = (3,300\sqrt{42} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 31,047 \text{ MPa}$$

l_c = 4 m = 4,000 mm

For the Top Column (Above):

$$\frac{t_b}{t_a} = \frac{130}{238} = 0.55$$

$$\frac{H}{H_c} = \frac{4,000}{3,632} = 1.10$$

Thus, $k_{BA} = 4.88$ and $C_{BA} = 0.6$ by interpolation.

$$K_c = \frac{4.88 E_{cc} I_c}{\ell_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,top} = 4.88 \times 31,047 \times \frac{5.21 \times 10^9}{4,000 \times 1,000} = 1.97 \times 10^8 \text{ N.m}$$

c. Torsional stiffness of torsional members, K_t.

$$K_t = \frac{9 E_{cs} C}{\ell_t \left(1 - \frac{c_2}{\ell_t}\right)^3}$$ ***CSA A23.3-14(13.8.2.8)***

$$K_t = \frac{9 \times 29,002 \times 4.45 \times 10^9}{9,000 \times (1 - 500/9,000)^3 \times 1,000} = 1.53 \times 10^8 \text{ N.m}$$

Where $C = \sum \left(1 - 0.63\frac{x}{y}\right)\left(\frac{x^3 y}{3}\right)$ ***CSA A23.3-14(13.8.2.9)***

$$C = \left(1 - 0.63 \times \frac{368}{500}\right)\left(368^3 \times \frac{500}{3}\right) = 4.45 \times 10^9 \text{ mm}^4$$

$c_2 = 500$ mm, $\ell_2 = 9$ m $= 9,000$ mm

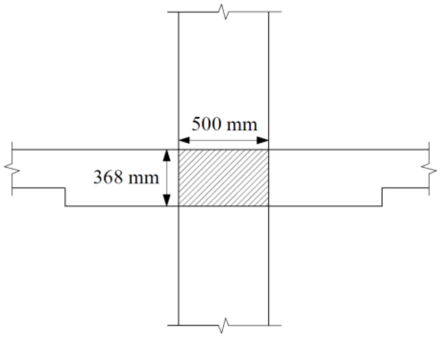
Equivalent column stiffness K_{ec}.

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

$$K_{ec} = \frac{(1.97 + 1.15)(2 \times 1.53)}{[(2.15 + 1.97) + (2 \times 1.53)]} \times 10^8$$

$$K_{ec} = 1.76 \times 10^8 \text{ N.m}$$

Where $\sum K_t$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.



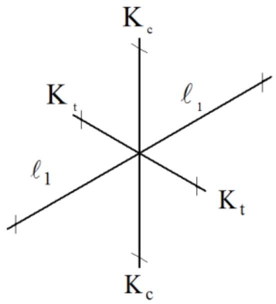


Figure 9 – Torsional Member

Figure 10 – Column and Edge of Slab

d. Slab-beam joint distribution factors, *DF*.

At exterior joint,

$$DF = \frac{2.36}{(1.76+2.36)} = 0.57$$

At interior joint,

$$DF = \frac{2.36}{(2.36+2.36+1.76)} = 0.36$$

COF for slab-beam =0.576

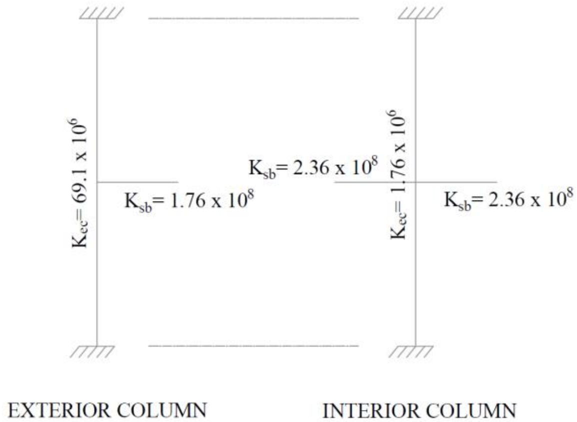


Figure 11 – Slab and Column Stiffness

2.1.3. Elastic frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ***CSA A23.3-14 (13.8.4.2)***

$$\frac{L}{D}=\frac{3}{(6.24+1)}=0.41<\frac{3}{4}$$

a. Factored load and Fixed-End Moments (FEM's).

For slab:

Factored dead load $w_{df}=1.25\times(6.24+1)=9.05\text{ kN/m}^2$

Factored live load $w_{lf}=1.5\times3=4.5\text{ kN/m}^2$

Factored load $w_f=w_{df}+w_{lf}=13.55\text{ kN/m}^2$

For drop panels:

Factored dead load $w_{df}=1.25\times(24\times0.108)=3.24\text{ kN/m}^2$

Factored live load $w_{lf}=1.5\times0=0\text{ kN/m}^2$

Factored load $w_f=w_{df}+w_{lf}=3.24\text{ kN/m}^2$

Fixed-end moment, $FEM=\Sigma_{i=1}^{n}m_{NFi}\times w_i\times l_1^2$ ***PCA Notes on ACI 318-11 (Table A1)***

$$FEM=0.0913\times13.55\times9\times9^2+0.0163\times3.24\times(9/6)\times9^2+0.0163\times3.24\times(9/3)\times9^2$$

$$FEM=914.9\text{ kN.m}$$

b. Moment distribution. Computations are shown in Table 1. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{f,midspan}=M_o-\frac{(M_{uL}+M_{uR})}{2}$$

Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example (6% difference compared with the exact value from spSlab).

Positive moment in span 1-2:

$$M_u=(13.55\times9)\frac{9^2}{8}+2\times\left[\frac{(3.24\times9/6)\times9/6}{2\times9}\times9/6\times\left(9-9/2\right)\right]-\frac{(428.6+1093.2)}{2}$$

$$M_f=479.3\text{ kN.m}$$

Table 1 - Moment Distribution for Equivalent Frame

Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.573	0.364	0.364	0.364	0.364	0.573
COF	0.576	0.576	0.576	0.576	0.576	0.576
FEM	914.9	-914.9	914.9	-914.9	914.9	-914.9
Dist	-523.8	0.0	0.0	0.0	0.0	523.8
CO	0.0	-301.8	0.0	0.0	301.8	0.0
Dist	0.0	109.9	109.9	-109.9	-109.9	0.0
CO	63.3	0.0	-63.3	63.3	0.0	-63.3
Dist	-36.3	23.1	23.1	-23.1	-23.1	36.3
CO	13.3	-20.9	-13.3	13.3	20.9	-13.3
Dist	-7.6	12.4	12.4	-12.4	-12.4	7.6
CO	7.2	-4.4	-7.2	7.2	4.4	-7.2
Dist	-4.1	4.2	4.2	-4.2	-4.2	4.1
CO	2.4	-2.4	-2.4	2.4	2.4	-2.4
Dist	-1.4	1.7	1.7	-1.7	-1.7	1.4
CO	1.0	-0.8	-1.0	1.0	0.8	-1.0
Dist	-0.6	0.7	0.7	-0.7	-0.7	0.6
CO	0.4	-0.3	-0.4	0.4	0.3	-0.4
Dist	-0.2	0.3	0.3	-0.3	-0.3	0.2
CO	0.1	-0.1	-0.1	0.1	0.1	-0.1
Dist	-0.1	0.1	0.1	-0.1	-0.1	0.1
CO	0.1	-0.1	-0.1	0.1	0.1	-0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M, kN.m	428.6	-1093.2	979.5	-979.5	1093.2	-428.6
Midspan M, kN.m	479.3		260.8		473.9	

2.1.4. Factored moments used for Design

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 12. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than 0.175 l_1 from the centers of supports. ***CSA A23.3-14 (13.8.5.1)***

$$500 \text{ mm} < 0.175 \times 9{,}000 = 1{,}575 \text{ mm (use face of supporting location)}$$

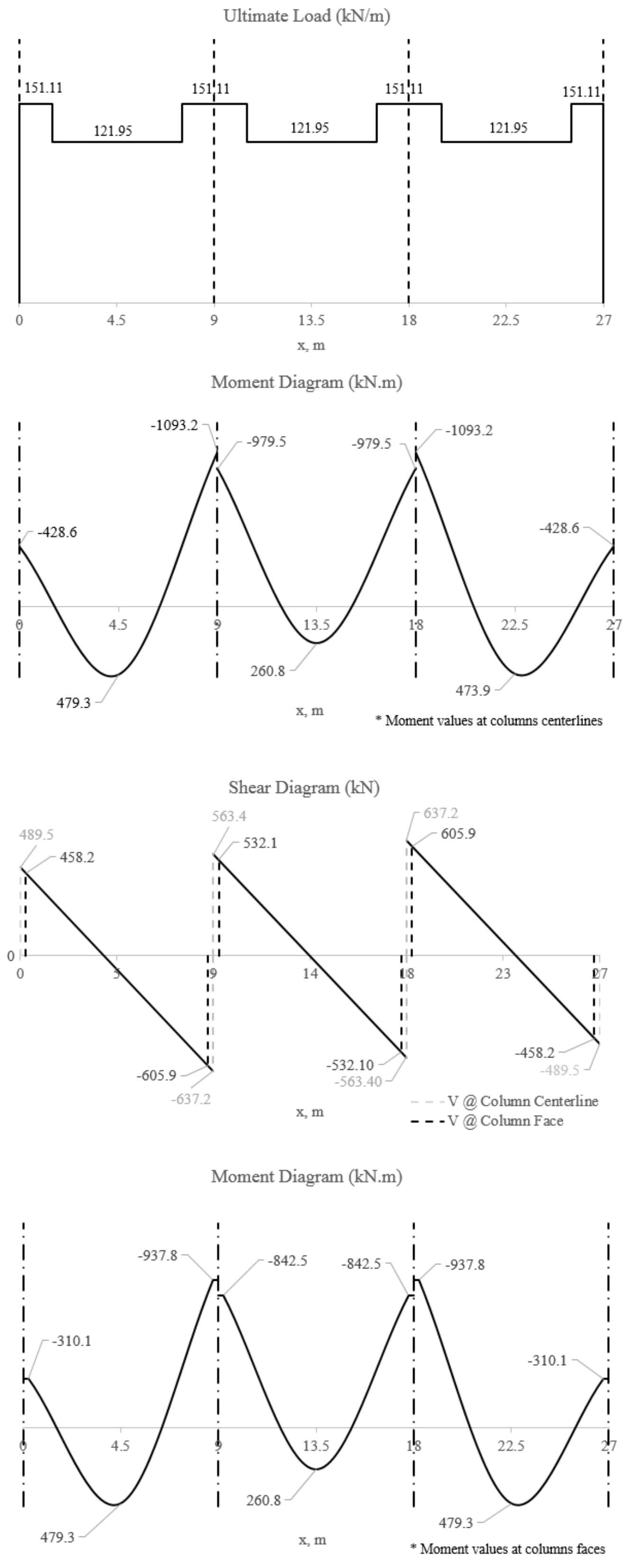


Figure 12 - Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

2.1.5. Distribution of design moments

Check Applicability of Direct Design Method:

1. There is a minimum of three continuous spans in each direction. ***CSA A23.3-14 (13.9.1.2)***

2. Successive span lengths are equal. ***CSA A23.3-14 (13.9.1.3)***

3. Loads are uniformly distributed over the entire panel ***CSA A23.3-14 (13.9.1.4)***

4. Factored live-to-dead load ratio of 0.5 < 2.0 ***CSA A23.3-14 (13.9.1.4)***

(Note: The self-weight of the drop panels is not uniformly distributed entirely along the span. However, the variation in load magnitude is small).

After the negative and positive moments have been determined for the slab-beam strip, the ACI code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM. ***CSA A23.3-14 (13.11.2.2)***

Distribution of factored moments at critical sections is summarized in Table 2.

Table 2 - Distribution of factored moments

		Slab-beam Strip	Column Strip		Middle Strip	
		Moment (kN.m)	Percent	Moment (kN.m)	Percent	Moment (kN.m)
End Span	Exterior Negative	310.09	100	310.09	0	0.00
	Positive	479.32	60	287.59	40	191.73
	Interior Negative	937.84	82.5	773.72	17.5	164.12
Interior Span	Negative	842.53	82.5	695.09	17.5	147.44
	Positive	260.75	60	156.45	40	104.30

2.1.6. Flexural reinforcement requirements

a. Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – exterior negative location is provided below.

$M_f = 310.09 \text{ kN.m}$

Use d_{avg} = 332 mm

In this example, jd is assumed equal to $0.98d$. The assumption will be verified once the area of steel in finalized.

Assume $jd = 0.98 \times d = 325.4 \text{ mm}$

Column strip width, $b = 9{,}000 / 2 = 4{,}500 \text{ mm}$

Middle strip width, $b = 9{,}000 - 4{,}500 = 4{,}500 \text{ mm}$

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{310.9 \times 10^6}{0.85 \times 400 \times 325.4} = 2{,}803 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.80 > 0.67$ **_CSA A23.3-14 (10.1.7)_**

Recalculate $'a'$ for the actual $A_s = 2803\ \text{mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85 \times 2834 \times 400}{0.65 \times 0.80 \times 35 \times 4,500} = 11.67\ \text{mm}$

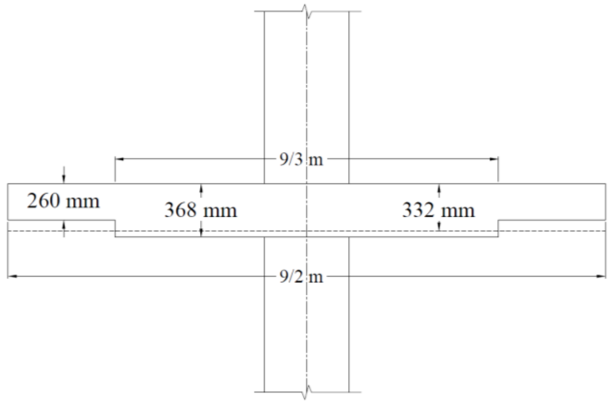
$jd = d - a/2 = 0.98d$

Therefore, the assumption that *jd* equals to *0.98d* is valid.

The slab have two thicknesses in the column strip (368 mm for the slab with the drop panel and 260 mm for the slab without the drop panel).

The weighted slab thickness:

$$h_w = \frac{368 \times (9/3) + 260 \times (9/2 - 9/3)}{(9/3) + (9/2 - 9/3)} = 332\ \text{mm}$$



$A_{s,\min} = 0.002 \times 4500 \times 332 = 2988\ \text{mm}^2 > 2814\ \text{mm}^2$ **_CSA A23.3-14 (7.8.1)_**

Maximum spacing: **_CSA A23.3-14 (13.10.4)_**

- Negative reinforcement in the band defined by b_b: $1.5h_w = 498\ \text{mm} \leq 250\ \text{mm}$
- Remaining negative moment reinforcement: $3h_w = 996\ \text{mm} \leq 500\ \text{mm}$

For the negative reinforcements at the exterior span the maximum spacing is 250 mm. To distribute the bars uniformly, the same minimum spacing is applied to the remaining negative moment reinforcements within the column.

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width b_b. Temperature and shrinkage reinforcement determined as specified in Clause 7.8.1 shall be provided in that section of the slab outside if the band region defined by b_b.

CSA A23.3-14 (13.10.3)

$b_b = 500 + 2 \times (1.5 \times 368) = 1,604\ \text{mm}$

Provide 10 – 15M bars with A_s = 3,000 mm^2 within b_b width and 9 – 15M bars with A_s = 1,800 mm^2 out of the band. Based on the procedure outlined above, values for all span locations are given in Table 3.

Table 3 - Required Slab Reinforcement for Flexure [Equivalent Frame Method (EFM)]								
Span Location		**M_u (kN.m)**	**b (mm)**	**d (mm)**	**A_s Req'd for flexure (mm^2)**	**Min A_s (mm^2)**	**Reinforcement Provided**	**A_s Prov. for flexure (mm^2)**
End Span								
Column Strip	Exterior Negative	310.10	4,500	332	2,803	2,988	24-15M*	4,800
	Positive	287.60	4,500	224	3,933	2,340	21-15M	4,200
	Interior Negative	773.72	4,500	332	7,204	2,988	37-15M	7,400
Middle Strip	Exterior Negative	0.0	4,500	224	0.0	2,340	12-15M*	2,400
	Positive	191.7	4,500	224	2,579	2,340	14-15M	2,800
	Interior Negative	164.12	4,500	224	2,207	2,340	12-15M*	2,400
Interior Span								
Column Strip	Positive	156.45	180	224	2,096	2340	12-15M*	2,400
Middle Strip	Positive	104.30	180	224	1,387	2340	12-15M*	2,400
* Design governed by minimum reinforcement								

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

The factored slab moment resisted by the column ($\gamma_f \times M_f$) shall be assumed to be transferred by flexure. When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of unbalanced moment given by γ_f shall be transferred by flexural reinforcement placed within a width b_b. ***CSA A23.3-14 (13.10.2)***

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_r$

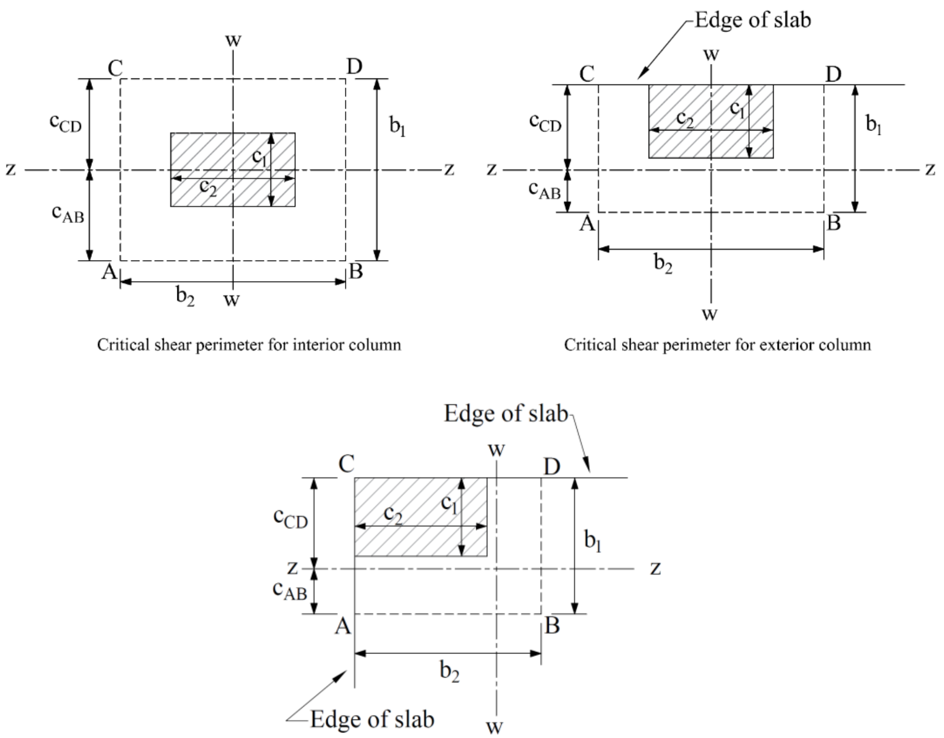
$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ ***CSA A23.3-14 (13.10.2)***

Where

b_1 = Width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see the following figure).

b_2 = Width of the critical section for shear measured in the direction perpendicular to b1 according to CSA A23.3-14, clause 13 (see the following figure).

b_b = Effective slab width = $c_2 + 3\times h_s$ ***CSA A23.3-14 (3.2)***



Critical shear perimeter for interior column

Critical shear perimeter for exterior column

Critical shear perimeter for corner column

Figure 13 – Critical Shear Perimeters for Columns

For exterior support:

d = 332 mm

$b_1 = c_1 + d/2$ = 500+ 332/2 = 666 mm

$b_2 = c_2 + d$ = 500 + 332 = 832 mm

$b_b = 500 + 3 \times 368 = 1,604$ mm

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{666/832}} = 0.626$$

$$\gamma_f\ M_{sc} = 0.626 \times 428.6 = 268.4 \text{ kN.m}$$

Using the same procedure in 2.1.6.a, the required area of steel:

$A_s = 2,412\ \text{mm}^2$

The area of steel provided to resist the flexural moment within the effective slab width b_b:

$A_{s,provided} = 3{,}000\ \text{mm}^2$

Then, there is no need to add additional reinforcement for the unbalanced moment.

Based on the procedure outlined above, values for all supports are given in Table 4.

Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column (EFM)									
Span Location		**M_{sc}* (kN.m)**	**γ_f**	**γ_f M_{sc} (kN.m)**	**Effective slab width, b_b (mm)**	**d (mm)**	**A_s req'd within b_b (mm^2)**	**A_s prov. For flexure within b_b (mm^2)**	**Add'l Reinf.**
End Span									
Column Strip	Exterior Negative	428.6	0.626	268.4	1,604	332	2,412	3,000	-
	Interior Negative	113.8	0.60	68.3	1,604	332	607	3,000	-
*M_{sc} is taken at the centerline of the support in Equivalent Frame Method solution.									

2.1.7. Factored moments in columns

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Referring to Figure 12, the unbalanced moment at the exterior and interior joints are:

Exterior Joint = +428.6 kN.m

Joint 2= -1,093.2 + 979.5 = -113.7 kN.m

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced slab moments (M_{sc}) to the exterior and interior columns are shown in the following figure.

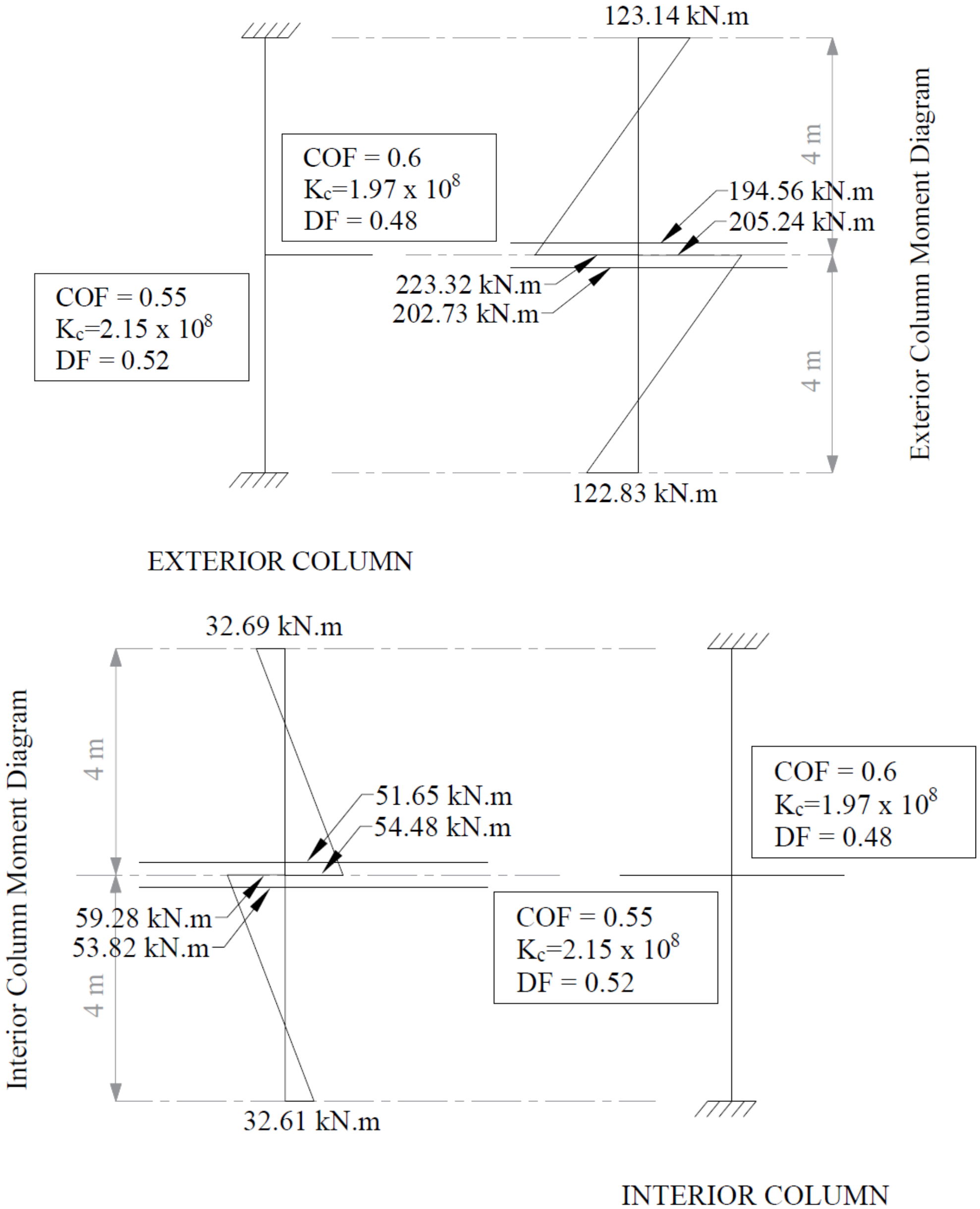


Figure 14 - Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

For Top column (Above):	For Bottom column (Below):
$M_{col,Exterior}$= 194.56 kN.m	$M_{col,Exterior}$= 202.73 kN.m
$M_{col,Interior}$ = 51.65 kN.m	$M_{col,Interior}$ = 53.82 kN.m

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. The moment values at the face of interior, exterior, and corner columns from the unbalanced moment values are shown in the following table.

Table 5 – Factored Moments in Columns			
M_u kN.m	**Column Location**		
	Interior	**Exterior**	**Corner**
M_{ux}	54.57	205.5	117.17
M_{uy}	54.57	54.57	117.17

3. Design of Columns by spColumn

This section includes the design of interior, edge, and corner columns using spColumn software. The preliminary dimensions for these columns were calculated previously in section one.

3.1. Determination of factored loads

Interior Column:

Assume 5 story building

Tributary area for interior column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = 9 \times 9 = 81 \text{ m}^2$$

Tributary area for interior column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = 3 \times 3 = 9 \text{ m}^2$$

Assuming five story building

$$P_f = n \times w_f \times A_{Tributary} = 5 \times (13.55 \times 81 + 9 \times 3.24) = 5{,}633.5 \text{ kN}$$

$M_{f,x}$ = 54.57 kN.m (see the previous Table)

$M_{f,y}$ = 54.57 kN.m (see the previous Table)

Edge (Exterior) Column:

Tributary area for edge column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{9}{2} + \frac{0.5}{2}\right) \times 9 = 42.75 \text{ m}^2$$

Tributary area for edge column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left(\frac{3}{2} + \frac{0.5}{2}\right) \times 3 = 5.25 \text{ m}^2$$

$$P_f = n \times w_f \times A_{Tributary} = 5 \times (13.55 \times 42.75 + 3.24 \times 5.25) = 2{,}981.4 \text{ kN}$$

$M_{f,x}$ = 205.5 kN.m (see the previous Table)

$M_{f,y}$ = 54.57 kN.m (see the previous Table)

Corner Column:

Tributary area for corner column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{9}{2}+\frac{0.5}{2}\right)\times\left(\frac{9}{2}+\frac{0.5}{2}\right) = 22.56 \text{ m}^2$$

Tributary area for corner column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left(\frac{3}{2}+\frac{0.5}{2}\right)\times\left(\frac{3}{2}+\frac{0.5}{2}\right) = 3.06 \text{ m}^2$$

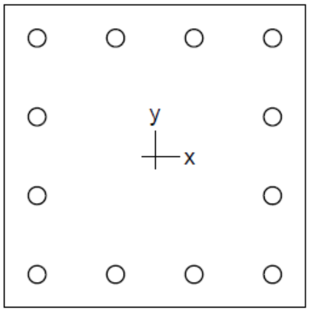
$$P_f = n\times w_f \times A_{Tributary} = 5\times(13.55\times 22.56 + 3.24\times 3.24) = 1{,}578.2 \text{ kN}$$

$M_{u,x}$ = 117.2 kN.m   (see the previous Table)

$M_{u,y}$ = 117.2 kN.m   (see the previous Table)

3.2. Moment Interaction Diagram

Interior Column:



500 x 500 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

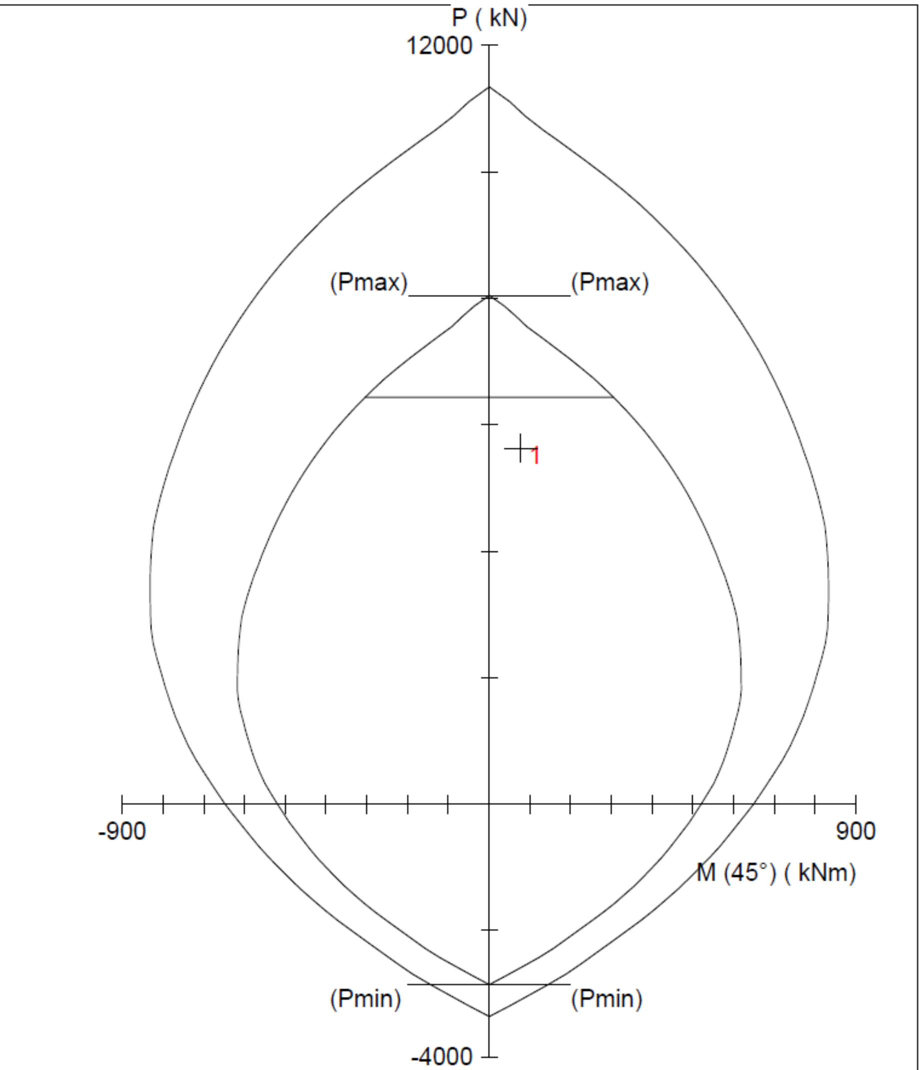
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 07/05/18

Time: 11:42:15



STRUCTUREPOINT - spColumn v6.00 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Column Design\Interior Column.col

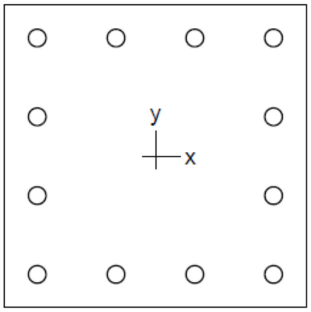
Project: Two Way Slab With Drop Panels

Column: Interior		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 250000 mm^2	12 #30 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 8400 mm^2	rho = 3.36%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 5.21e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 5.21e+009 mm^4
Beta1 = 0.865		Min clear spacing = 100 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 500 mm

<u>Edge Column:</u>



500 x 500 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

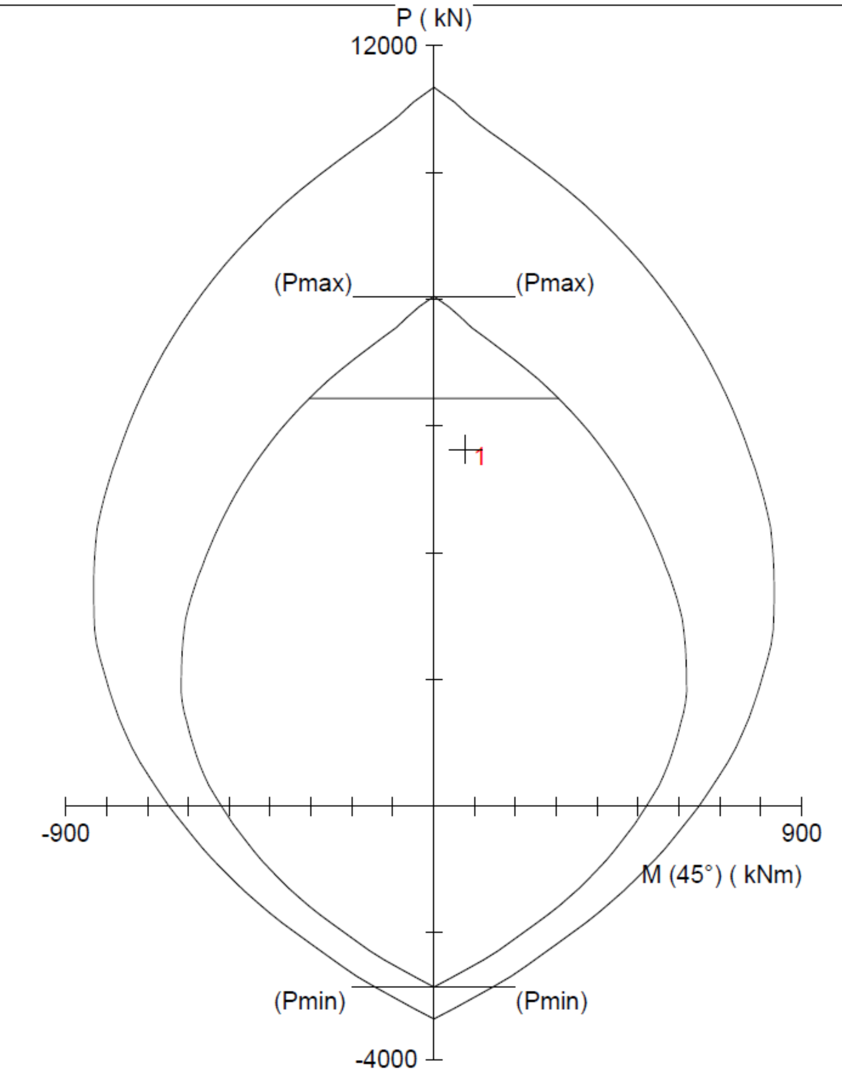
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 07/05/18

Time: 11:44:55



STRUCTUREPOINT - spColumn v6.00 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: c:\tsda\two-way flate slab floor with drop panel (csa)\column design\exterior column.col

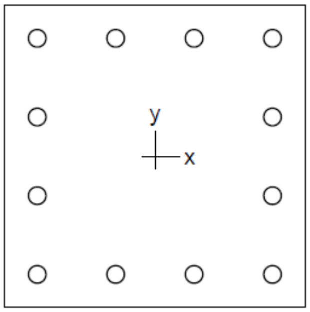
Project: Two Way Slab With Drop Panels

Column: Interior		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 250000 mm^2	12 #30 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 8400 mm^2	rho = 3.36%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 5.21e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 5.21e+009 mm^4
Beta1 = 0.865		Min clear spacing = 100 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 500 mm

Corner Column:



500 x 500 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

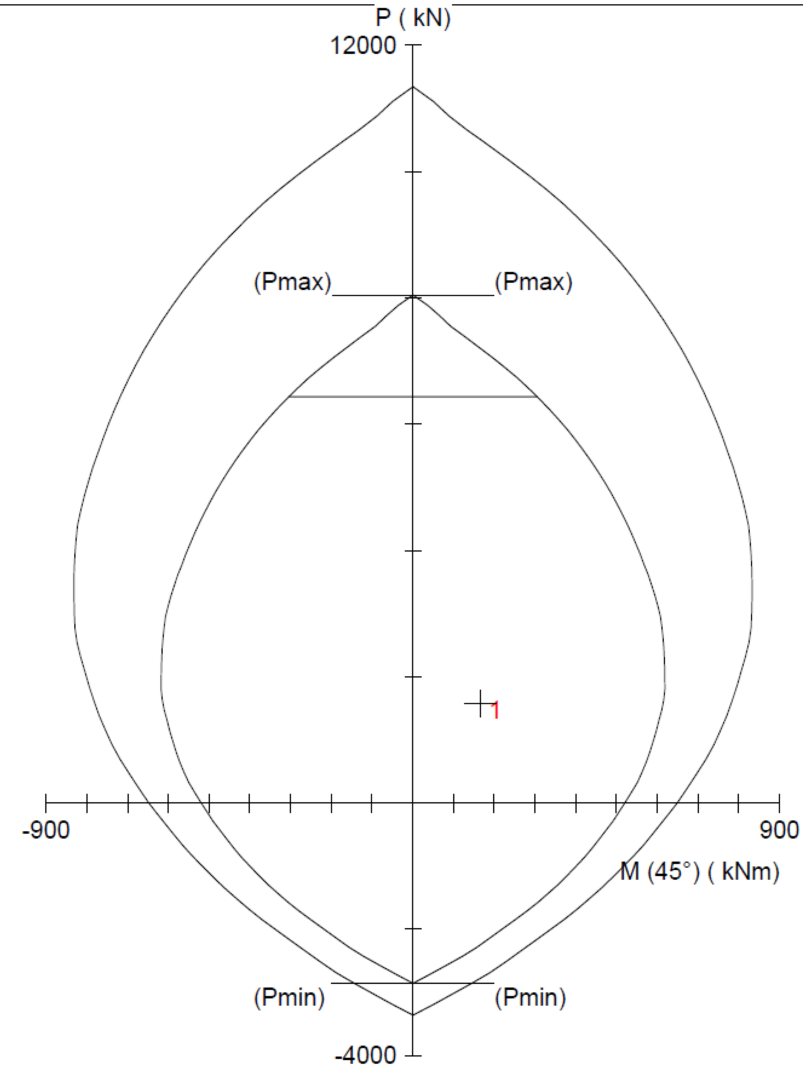
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 07/05/18

Time: 11:45:50



STRUCTUREPOINT - spColumn v6.00 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: c:\tsda\two-way flate slab floor with drop panel (csa)\column design\corner column.col

Project: Two Way Slab With Drop Panels

Column: Interior		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 250000 mm^2	12 #30 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 8400 mm^2	rho = 3.36%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 5.21e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 5.21e+009 mm^4
Beta1 = 0.865		Min clear spacing = 100 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 500 mm

4. Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with CSA A23.3-14 Chapter 13.

4.1. One-Way (Beam action) Shear Strength

CSA A23.3-14 (13.3.6)

One-way shear is critical at a distance *d* from the face of the column as shown in Figure 3. Figures 15 and 16 show the factored shear forces (V_f) at the critical sections around each column and each drop panel, respectively. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$V_r = V_c + V_s + V_p = V_c \quad , \quad (V_s = V_p = 0)$$

CSA A23.3-14 (Eq. 11.4)

Where:

$$V_c = \varphi_c \lambda \beta \sqrt{f_c^{'}} b_w d_v$$

CSA A23.3-14 (Eq. 11.5)

Note: The calculations below follow one of two possible approaches for checking one-way shear. Refer to the conclusions section for a comparison with the other approach.

4.1.1. At distance d_v from the supporting column

$$h_{weighted} = \frac{368 \times 9/3 + 260 \times (9 - 9/3)}{9} = 296 \text{ mm}$$

$$d_w = 296 - 28 - 16/2 = 260 \text{ mm}$$

$$d_v = Max\ (0.9d, 0.72h) = Max\ (0.9 \times 260, 0.72 \times 260) = 234 \text{ mm}$$

CSA A23.3-14 (3.2)

$\lambda = 1$ for normal weight concrete

$$\beta = \frac{230}{(1,000 + d_v)} = \frac{230}{(1,000 + 234)} = 0.186$$

CSA A23.3-14 (11.3.6.3)

$$V_c = 0.65 \times 1 \times 0.186 \times \sqrt{35} \times 9,000 \times \frac{234}{1,000} = 1,506.4 \text{ kN} > V_f$$

Because $V_r \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

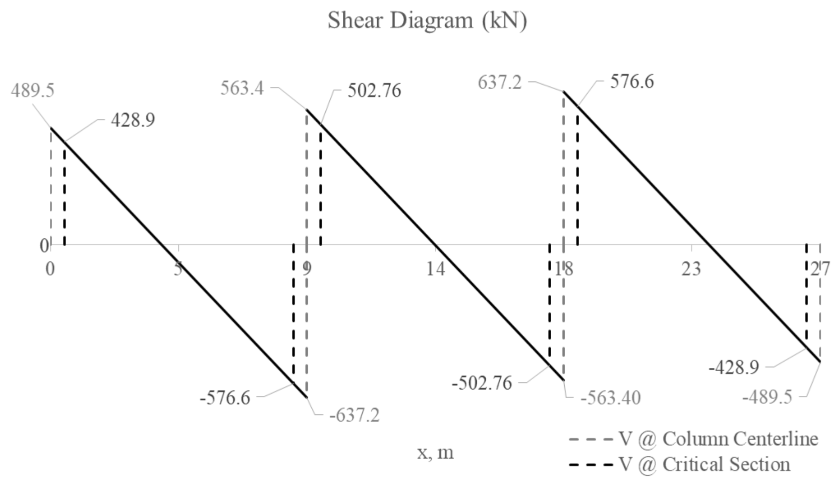


Figure 15 – One-way shear at critical sections (at distance *d* from the face of the supporting column)

4.1.2. At the face of the drop panel

$h = 260 \text{ mm}$

$d \;= 260 - 28 - 16/2 = 224 \text{ mm}$

$d_v = Max\ (0.9d, 0.72h) = Max\ (0.9 \times 224, 0.72 \times 260) = 202 \text{ mm}$ ***CSA A23.3-14 (3.2)***

$\lambda = 1$ for normal weight concrete

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{35} \times 9,000 \times \frac{202}{1,000} = 1,465.2 \text{ kN} > V_f$$

Because $V_r \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

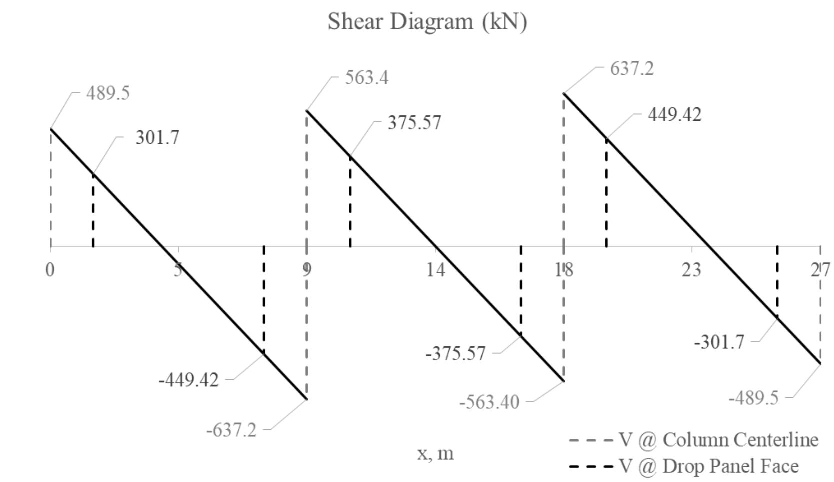


Figure 16 – One-way shear at critical sections (at the face of the drop panel)

4.2. Two-Way (Punching) Shear Strength **CSA A23.3-14 (13.3.2)**

4.2.1. Around the columns faces

Two-way shear is critical on a rectangular section located at $d/2$ away from the face of the column as shown in Figure 13.

a. Exterior column:

The factored shear force (V_f) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section ($d/2$ away from column face).

$$V_f = V - w_f\left(b_1 \times b_2\right) = 474.9 - 16.79\left(\frac{666 \times 832}{10^6}\right) = 465.6 \text{ kN}$$

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$M_{unb} = M - V_f\left(b_1 - c_{AB} - c_1/2\right) = 428.5 - 474.9\left(\frac{666 - 205 - 500/2}{10^3}\right) = 330.3 \text{ kN.m}$$

For the exterior column in Figure 13, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\textit{moment of area of the sides about AB}}{\textit{area of the sides}} = \frac{2(666 \times (832 - 666) \times 666/2)}{2 \times 666 \times 332 + 832 \times 332} = 205 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + \left(b_1 d\right)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$J = 2\left(\frac{666 \times 332^3}{12} + \frac{332 \times 666^3}{12} + \left(666 \times 332\right)\left(\frac{666}{2} - 205\right)^2\right) + 832 \times 332 \times 205^2$$

$$J = 3.93 \times 10^{10} \text{ mm}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.626 = 0.374$$ **CSA A23.3-14 (Eq. 13.8)**

The length of the critical perimeter for the exterior column:

$$b_o = 2 \times 666 + 832 = 2{,}164 \text{ mm}$$

The two-way shear stress (v_f) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} c_{AB}}{J}$$ **CSA A23.3-14 (Eq.13.9)**

$$v_f = \frac{465.6 \times 1,000}{2,164 \times 332} + \frac{0.374 \times (330.3 \times 10^6) \times 205}{2.926 \times 10^{10}} = 1.29 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: **CSA A23.3-14 (13.3.4.1)**

a) $$v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{1}\right) 0.19 \times 0.65 \times \sqrt{35} = 2.19 \text{ MPa}$$

b) $$v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{3 \times 332}{2164} + 0.19\right) \times 1 \times 0.65 \times \sqrt{35} = 2.5 \text{ MPa}$$

c) $$v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa}$$

If the effective depth. D, used in two-way shear calculations exceeds 300 mm, the value of v_c obtained shall be multiplied by 1,300/ (1,000+d). **CSA A23.3-14 (13.3.4.1)**

$$v_r = v_c = 1.46 \times \frac{1,300}{(1,000 + 332)} = 1.426 \text{ MPa}$$

Since $v_c \geq v_f$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

b. Interior column:

$$V_f = V - w_f (b_1 \times b_2) = 622.6 + 548.8 - 16.79 \left(\frac{832 \times 832}{10^6}\right) = 1,159.8 \text{ kN}$$

$$M_{unb} = M - V_f (b_1 - c_{AB} - c_1 / 2) = 113.8 - 1,159.8(0) = 113.8 \text{ kN}$$

For the interior column in Figure 13, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{832}{2} = 416 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 b_2 d c_{AB}^2$$

$$J = 2\left(\frac{832 \times 332^3}{12} + \frac{332 \times 832^3}{12} + (832 \times 332)\left(\frac{832}{2} - 416\right)^2\right) + 2 \times 832 \times 332 \times 416^2$$

$$J = 1.33 \times 10^{11} \text{ mm}^4$$

$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$ **CSA A23.3-14 (Eq. 13.8)**

The length of the critical perimeter for the interior column:

$$b_o = 2 \times (832 + 832) = 3{,}328 \text{ mm}$$

The two-way shear stress (v_f) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} c_{AB}}{J}$$ **CSA A23.3-14 (Eq.13.9)**

$$v_f = \frac{1{,}171.4 \times 1{,}000}{3{,}328 \times 332} + \frac{0.4 \times (113.8 \times 10^6) \times 416}{1.33 \times 10^{11}} = 1.19 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: **CSA A23.3-14 (13.3.4.1)**

a) $v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{1}\right) 0.19 \times 0.65 \times \sqrt{35} = 2.19 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{4 \times 332}{3328} + 0.19\right) \times 1 \times 0.65 \times \sqrt{35} = 2.27 \text{ MPa}$

c) $v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa}$

If the effective depth. D, used in two-way shear calculations exceeds 300 mm, the value of v_c obtained shall be multiplied by 1,300/ (1,000+d). **CSA A23.3-14 (13.3.4.1)**

$$v_r = v_c = 1.46 \times \frac{1{,}300}{(1{,}000 + 332)} = 1.426 \text{ Mpa}$$

Since $v_c \geq v_f$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

c. Corner column:

In this example, interior equivalent frame strip was selected where it only have exterior and interior supports (no corner supports are included in this strip). However, the two-way shear strength of corner supports usually governs. Thus, the two-way shear strength for the corner column in this example will be checked for educational purposes. Same procedure is used to find the reaction and factored unbalanced moment used for shear transfer at the centroid of the critical section for the corner support for the exterior equivalent frame strip.

$$V_f = V - w_f (b_1 \times b_2) = 261.4 - 16.79 \left(\frac{666 \times 666}{10^6}\right) = 253.9 \text{ kN}$$

$$M_{unb} = M - V_f (b_1 - c_{AB} - c_1 / 2) = 247.7 - 253.9 \left(\frac{666 - 166.5 - 500/2}{10^3}\right) = 184.3 \text{ kN.m}$$

For the interior column in Figure 13, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{moment\ of\ \ area\ of\ \ the\ sides\ about\ AB}{area\ of\ \ the\ sides} = \frac{(666\times332\times666/2)}{666\times332+666\times332} = 166.5 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J = \left(\frac{b_1d^3}{12}+\frac{db_1^3}{12}+(b_1d)\left(\frac{b_1}{2}-c_{AB}\right)^2\right)+b_2dc_{AB}^2$$

$$J_c = \left(\frac{666\times332^3}{12}+\frac{666\times332^3}{12}+(666\times332)\left(\frac{666}{2}-166.5\right)^2\right)+666\times332\times166.5^2$$

$$J_c = 2.25\times10^{10} \text{ mm}^4$$

$$\gamma_v = 1-\gamma_f = 1-0.600 = 0.400$$ **CSA A23.3-14 (Eq. 13.8)**

The length of the critical perimeter for the corner column:

$$b_o = 666+666 = 1,332 \text{ mm}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_f = \frac{V_f}{b_o\times d}+\frac{\gamma_v M_{unb} c_{AB}}{J}$$ **CSA A23.3-14 (Eq.13.9)**

$$v_f = \frac{253.9\times1,000}{1,332\times332}+\frac{0.4\times(184.3\times10^6)\times166.5}{2.25\times10^{10}} = 1.12 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: **CSA A23.3-14 (13.3.4.1)**

a) $v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1}\right)0.19\times0.65\times\sqrt{35} = 2.19 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{2\times332}{1,332}+0.19\right)\times1\times0.65\times\sqrt{35} = 2.65 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{35} = 1.46 \text{ MPa}$

If the effective depth. D, used in two-way shear calculations exceeds 300 mm, the value of v_c obtained shall be multiplied by 1300/ (1000+d). **CSA A23.3-14 (13.3.4.1)**

$$v_r = v_c = 1.46\times\frac{1,300}{(1,000+332)} = 1.426 \text{ MPa}$$

Since $v_c \geq v_f$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

4.2.2. Around drop panels

Two-way shear is critical on a rectangular section located at d/2 away from the face of the drop panel.

Note: The two-way shear stress calculations around drop panels do not have the term for unbalanced moment since drop panels are a thickened portion of the slab and are not considered as a support.

a. Exterior drop panel:

$$V_f = V - w_f A = 474.9 - 13.55\left(\frac{1,862\times 3,224}{10^6}\right) = 393.6 \text{ kN}$$

The length of the critical perimeter for the exterior drop panel:

$$b_o = 2\times 1,862 + 3,224 = 6,948 \text{ mm}$$

The two-way shear stress (v_f) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d}$$ ***CSA A23.3-14 (N.13.3.5.4)***

$$v_f = \frac{393.6\times 1,000}{6,948\times 224} = 0.25 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: ***CSA A23.3-14 (13.3.4.1)***

a) $v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1}\right)0.19\times 0.65\times\sqrt{35} = 2.19 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{3\times 224}{6,948}+0.19\right)\times 1\times 0.65\times\sqrt{35} = 1.10 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times 1\times 0.65\times\sqrt{35} = 1.46 \text{ MPa}$

Since $v_c \geq v_f$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

b. Interior drop panel:

$$V_f = V - w_f A$$

$$V_f = 622.6 + 548.8 - 13.55\left(\frac{3,224\times 3,224}{10^6}\right) = 1,030.6 \text{ kN}$$

The length of the critical perimeter for the interior drop panel:

$$b_o = 2\times(3,224 + 3,224) = 12,896 \text{ mm}$$

The two-way shear stress (v_f) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d}$$ ***CSA A23.3-14 (N.13.3.5.4)***

$$v_f = \frac{1,030.6 \times 1,000}{12,896 \times 224} = 0.36 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: ***CSA A23.3-14 (13.3.4.1)***

a) $v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{1}\right) 0.19 \times 0.65 \times \sqrt{35} = 2.19 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{4 \times 224}{12,896} + 0.19\right) \times 1 \times 0.65 \times \sqrt{35} = 1.00 \text{ MPa}$

c) $v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa}$

Since $v_c \geq v_f$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

c. Corner drop panel:

$$V_f = V - w_f A$$

$$V_f = 261.4 - 13.55 \left(\frac{1,862 \times 1,862}{10^6}\right) = 214.4 \text{ kN}$$

The length of the critical perimeter for the corner drop panel:

$$b_o = 1,862 + 1,862 = 3724 \text{ mm}$$

The two-way shear stress (v_f) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d}$$ ***CSA A23.3-14 (N.13.3.5.4)***

$$v_f = \frac{214.4 \times 1,000}{3,724 \times 224} = 0.24 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: ***CSA A23.3-14 (13.3.4.1)***

a) $v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{1}\right) 0.19 \times 0.65 \times \sqrt{35} = 2.19 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{2 \times 224}{3,724} + 0.19\right) \times 1 \times 0.65 \times \sqrt{35} = 1.19 \text{ MPa}$

c) $v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{35} = 1.46 \text{ MPa}$

Since $v_c \geq v_f$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

5. Serviceability Requirements (Deflection Check)

Since the slab thickness was selected below the minimum slab thickness equations in CSA A23.3-14, the deflection calculations of immediate and time-dependent deflections are required and shown below including a comparison with spSlab model results.

5.1. Immediate (Instantaneous) Deflections

When deflections are to be computed, deflections that occur immediately on application of load shall be computed by methods or formulas for elastic deflections, taking into consideration the effects of cracking and reinforcement on member stiffness. Unless deflections are determined by a more comprehensive analysis, immediate deflection shall be computed using elastic deflection equations. ***CSA A23.3-14 (9.8.2.2 & 9.8.2.3)***

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

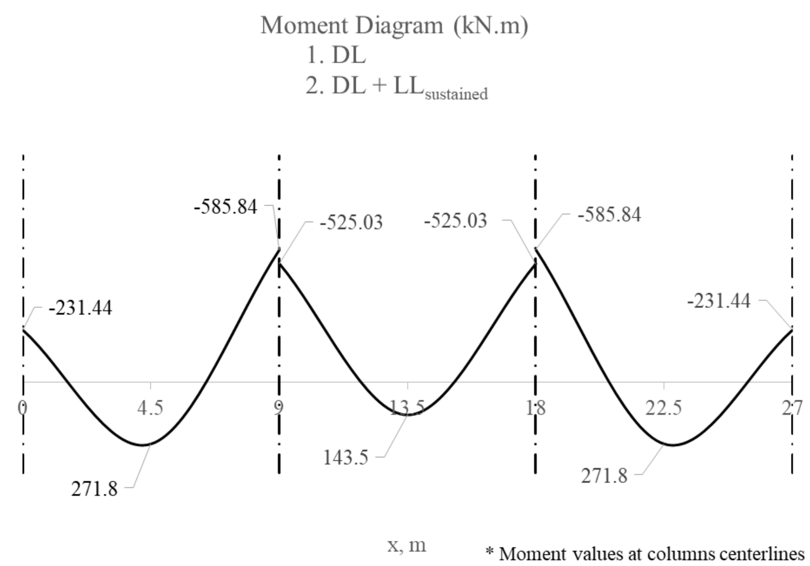
The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g$$ ***CSA A23.3-14 (Eq.9.1)***

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 17.



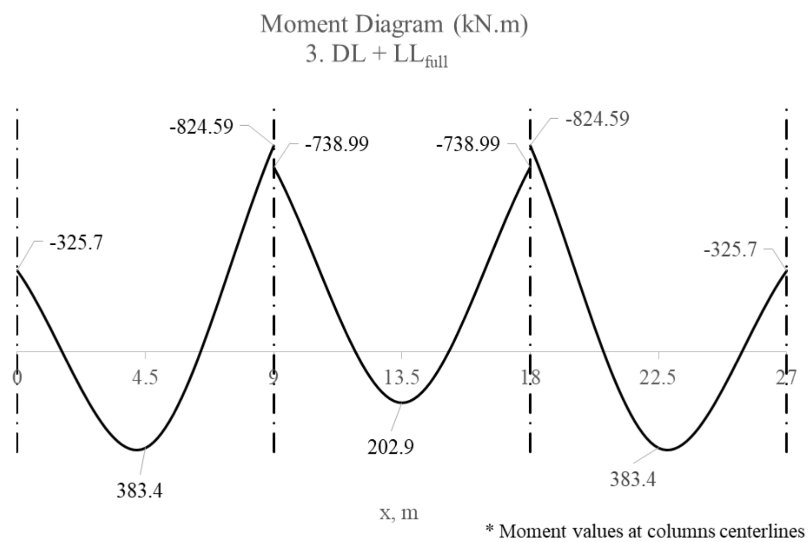


Figure 17 – Maximum Moments for the Three Service Load Levels
(No live load is sustained in this example)

For positive moment (midspan) section:

M_{cr} = Cracking moment.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(3.55/2)\times(1.32\times10^{10})}{130}\times10^{-6} = 179.9 \text{ kN.m}$$ **CSA A23.3-14 (Eq.9.2)**

f_r = Modulus of rapture of concrete.

$$f_r = 0.6\lambda\sqrt{f_c'} = 0.6\times1.0\times\sqrt{35} = 3.55 \text{ MPa}$$ **CSA A23.3-14 (Eq.8.3)**

I_g = Moment of inertia of the gross uncracked concrete section.

$$I_g = \frac{l_2 h^3}{12} = \frac{9,000 \times 260^3}{12} = 1.32 \times 10^{10} \text{ mm}$$

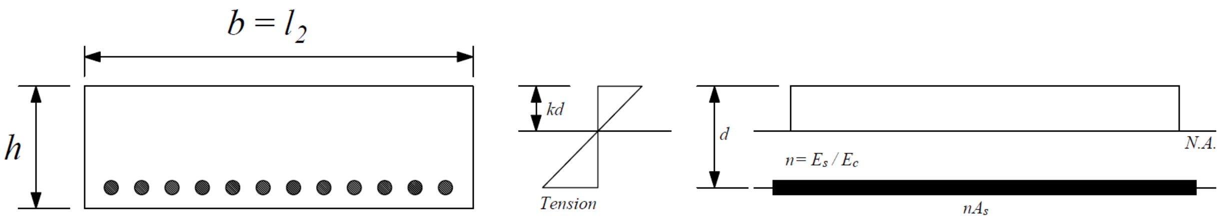
y_t = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, mm.

$$y_t = \frac{h}{2} = \frac{260}{2} = 130 \text{ mm}$$

I_{cr} = Moment of inertia of the cracked section transformed to concrete.

<u>CAC Concrete Design Handbook 4th Edition (Table 6.2(a))</u>

As calculated previously, the positive reinforcement for the end span frame strip is 35-15M bars are located along the section from the bottom of the slab. <u>Two of these bars are not continuous and will be conservatively excluded from the calculation of I_{cr} since they might not be adequately developed or tied (33 bars are used).</u> Figure 18 shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.



<u>Figure 18 – Cracked Transformed Section (positive moment section)</u>

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = (3,300\sqrt{f_c'} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{35} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 29,002 \text{ MPa}$$ ***<u>CSA A23.3-14(8.6.2.2)</u>***

$$n = \frac{E_s}{E_{cs}} = \frac{200,000}{29,002} = 6.9$$ ***<u>CAC Concrete Design Handbook 4th Edition (Table 6.2a)</u>***

$$B = \frac{b}{n\,A_s} = \frac{9,000}{6.9 \times (33 \times 200)} = 0.2 \text{ mm}^{-1}$$ ***<u>CAC Concrete Design Handbook 4th Edition (Table 6.2a)</u>***

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2 \times 224 \times 0.2 + 1}-1}{0.2} = 42.81 \text{ mm}$$

<u>CAC Concrete Design Handbook 4th Edition (Table 6.2a)</u>

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$ ***<u>CAC Concrete Design Handbook 4th Edition (Table 6.2a)</u>***

$$I_{cr} = \frac{9,000 \times (42.81)^3}{3} + 6.90 \times (33 \times 200) \times (224 - 42.81)^2 = 1.73 \times 10^9 \text{ mm}^4$$

For negative moment section (near the interior support of the end span):

The negative reinforcement for the end span frame strip near the interior support is 36-15M bars along the section from the top of the slab.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(3.55/2)\times(2.31\times10^{10})}{152.4}\times10^{-6} = 269.4 \text{ kN.m}$$ **CSA A23.3-14 (Eq.9.2)**

$$f_r = 0.6\lambda\sqrt{f_c^{'}} = 0.6\times1.0\times\sqrt{35} = 3.55 \text{ MPa}$$ **CSA A23.3-14 (Eq.8.3)**

$$I_g = 2.31\times10^{10} \text{ mm}^4$$

$$y_t = 152.4 \text{ mm}$$

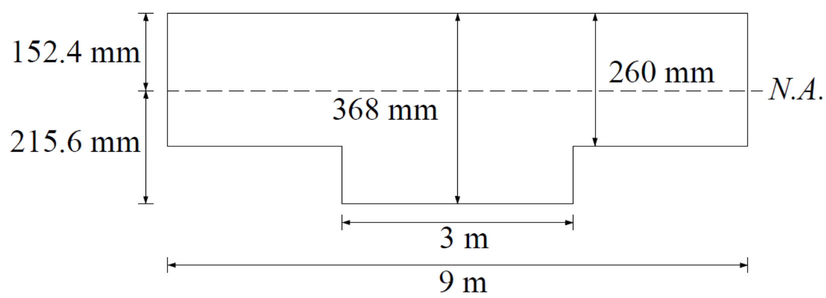


Figure 19 – I_g calculations for slab section near support

$$E_{cs} = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ **CSA A23.3-14(8.6.2.2)**

$$E_{cs} = (3,300\sqrt{35} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 29,002 \text{ MPa}$$

$$n = \frac{E_s}{E_{cs}} = \frac{200,000}{29,002} = 6.9$$ **CAC Concrete Design Handbook 4th Edition (Table 6.2a)**

$$B = \frac{b}{n\,A_s} = \frac{3,000}{6.9\times(36\times200)} = 0.06 \text{ mm}^{-1}$$ **CAC Concrete Design Handbook 4th Edition (Table 6.2a)**

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2\times332\times0.06+1}-1}{0.06} = 89.6 \text{ mm}$$

CAC Concrete Design Handbook 4th Edition (Table 6.2a)

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$ **CAC Concrete Design Handbook 4th Edition (Table 6.2a)**

$$I_{cr} = \frac{3,000\times(89.58)^3}{3} + 6.90\times(36\times200)\times(332-89.58)^2 = 3.64\times10^9 \text{ mm}^4$$

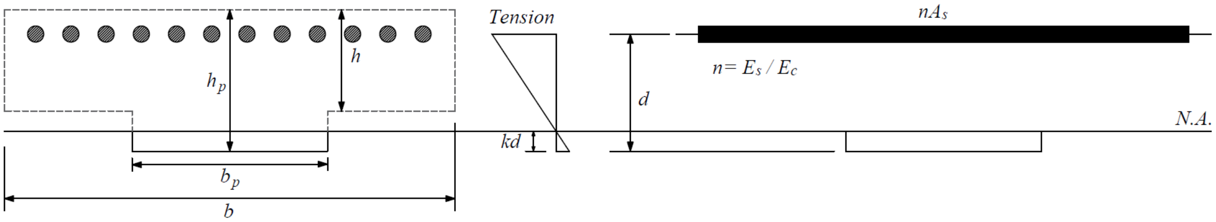


Figure 20 – Cracked Transformed Section (negative moment section)

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (9.1) unless obtained by a more comprehensive analysis.

For continuous prismatic members, the effective moment of inertia may be taken as the weighted average of the values obtained from Equation 9.1 for the critical positive and negative moment sections

CSA A23.3-14(9.8.2.4)

For the middle span (span with two ends continuous) with service load level ($D+LL_{full}$):

$$I_e^- = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , since } M_{cr} = 269.4 \text{ kN.m} < M_a = 739 \text{ kN.m}$$ ***CSA A23.3-14(Eq. 9.1)***

Where I_e^- is the effective moment of inertia for the critical negative moment section (near the support).

$$I_e^- = 4.64\times10^9 + \left(2.31\times10^{10} - 4.64\times10^9\right)\left(\frac{269.4}{739}\right)^3 = 5.53\times10^9 \text{ mm}^4$$

For the middle span (span with two ends continuous) with service load level ($D+LL_{full}$):

$$I_e^+ = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , since } M_{cr} = 179.9 \text{ kN.m} < M_a = 202.9 \text{ kN.m}$$ ***CSA A23.3-14(Eq. 9.1)***

$$I_e^+ = 1.31\times10^9 + \left(1.32\times10^{10} - 1.31\times10^9\right)\left(\frac{179.9}{202.9}\right)^3 = 9.59\times10^9 \text{ mm}^4$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. Both the midspan stiffness (I_e^+) and averaged span stiffness ($I_{e,avg}$) can be used in the calculation of immediate (instantaneous) deflection.

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70\, I_e^+ + 0.15\, \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for two ends continuous}$$ **CSA A23.3-14 (Eq.9.3)**

$$I_{e,avg} = 0.85\, I_e^+ + 0.15\, I_e^- \quad \text{for one end continuous}$$ **CSA A23.3-14 (Eq.9.4)**

However, these expressions lead to improved results only for continuous prismatic members. The drop panels in this example result in non-prismatic members and the following expressions should be used according to ACI 318-89:

$$I_{e,avg} = 0.50\, I_e^+ + 0.25\, \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ **ACI 435R-95 (2.14)**

For the middle span (span with two ends continuous) with service load level (D_l):

$$I_{e,avg} = 0.50 \times 13.18 \times 10^9 + 0.25\, \left(7.14 \times 10^9 + 7.14 \times 10^9\right) = 10.16 \times 10^9 \text{ mm}^4$$

$$I_{e,avg} = 0.50\, I_e^+ + 0.50\, I_e^- \text{ for end span}$$ **ACI 435R-95 (2.14)**

For the end span (span with one end continuous) with service load level (D_l):

$$I_{e,avg} = 0.50 \times 5.06 \times 10^9 + 0.50 \times 6.44 \times 10^9 = 5.75 \times 10^9 \text{ mm}^4$$

Where:

$I_{e,l}^-$ = The effective moment of inertia for the critical negative moment section near the left support.

$I_{e,l}^-$ = The effective moment of inertia for the critical negative moment section near the right support.

I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

Table 6 provides a summary of the required parameters and calculated values needed for deflections for exterior and interior spans.

Table 6 – Averaged Effective Moment of Inertia Calculations													
For Frame Strip													
Span	zone	I_g, mm^4 ($\times10^9$)	I_{cr}, mm^4 ($\times10^9$)	M_a, kN.m			M_{cr}, kN.m	I_e, mm^4 ($\times10^9$)			$I_{e,avg}$, mm^4 ($\times10^9$)		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	23.13	3.64	-231.4	-231.4	-325.7	269.4	23.13	23.13	14.67	5.75	5.75	4.10
	Midspan	13.18	1.73	271.7	271.7	383.4	179.9	5.06	5.06	2.91			
	Right	23.13	4.64	-585.8	-585.8	-824.6	269.4	6.44	6.44	5.28			
Int	Left	23.13	4.64	-525.0	-525.0	739	269.4	7.14	7.14	5.53	10.16	10.16	7.56
	Midspan	13.18	1.31	179.9	179.9	179.9	179.9	13.18	13.18	9.60			
	Right	23.13	4.64	-525.0	-525.0	739	269.4	7.14	7.14	5.53			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections in two-way slab systems, the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 21 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)

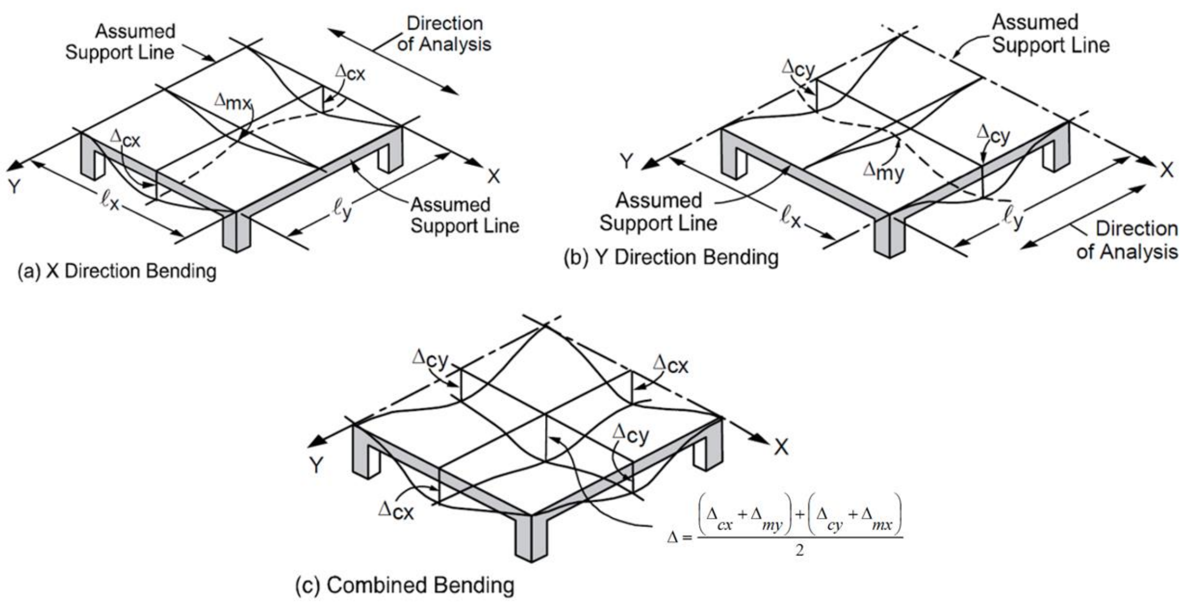


Figure 21 – Deflection Computation for a rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 22 shows the procedure of calculating the term Δ_{cx}. Same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \frac{I_{frame}}{I_c}$$

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)$$

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_R^-}{2}}{2}$$

$$\theta_{c,R} = \frac{M_{net,R}}{K_{ec}}$$

$$\theta_{c,L} = \frac{M_{net,L}}{K_{ec}}$$

$$\Delta_{frame,fixed} = \frac{wl^4}{384 E_c I_{frame}}$$

Figure 22 – Δ_{cx} calculation procedure

For end span - service dead load case:

$$\Delta_{frame,fixed} = \frac{wl^4}{384 E_c I_{frame,averaged}}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)

Where:

$\Delta_{frame,fixed}$ = Deflection of column strip assuming fixed end condition.

$$w = (1 + 24 \times 0.260)(9) = 65.16 \text{ kN/m}$$

$$E_{cs} = (3{,}300\sqrt{f_c'} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5}$$

CSA A23.3-14(8.6.2.2)

$$E_{cs} = (3{,}300\sqrt{35} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 29{,}000 \text{ MPa}$$

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 6 = 5.75 x 10^9 mm^4

$$\Delta_{frame,fixed} = \frac{(65.16)(9 \times 10^3)^4}{384(29{,}000)(5.75 \times 10^9)} = 5.31 \text{ mm}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \frac{I_{frame,averaged}}{I_{c,g}}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)

For this example and like in the spSlab program, the effective moment of inertia at midspan will be used.

LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

For the end span, LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF^+) are 1.00, 0.825, and 0.60, respectively (From Table 2 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.6 + \dfrac{1.0 + 0.825}{2}}{2} = 0.756$$

$I_{c,g}$ = The gross moment of inertia (I_g) for the column strip for service dead load = 6.59 x 10^9 mm^4

$$\Delta_{c,\,fixed} = 0.756 \times 0.0995 \times \frac{13.18 \times 10^9}{6.59 \times 10^9} = 8.03 \text{ mm}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)***

Where:

$\theta_{c,L}$ = Rotation of the span left support.

$(M_{net,L})_{frame}$ = 231 kN-m = Net frame strip negative moment of the left support.

K_{ec} = effective column stiffness = 1.76 x 10^5 kN-m/rad (calculated previously).

$$\theta_{c,L} = \frac{231}{1.76 \times 10^5} = 0.00131 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)***

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$\left(\frac{I_g}{I_e}\right)_{frame}$ = Gross-to-effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00131 \times \frac{9000-500}{8} \times \frac{13.18 \times 10^9}{5.75 \times 10^9} = 3.20 \text{ mm}$$

$$\theta_{c,R} = \frac{\left(M_{net,R}\right)_{frame}}{K_{ec}} = \frac{6.08 \times 10^7}{1.76 \times 10^{11}} = 0.00035 \text{ rad}$$

Where

$\theta_{c,R}$ = Rotation of the end span right support.

$(M_{net,R})_{frame}$ = Net frame strip negative moment of the right support.

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} = 0.00035 \times \frac{9000-500}{8} \times \frac{13.18 \times 10^9}{5.75 \times 10^9} = 0.84 \text{ mm}$$

Where:

$\Delta\theta_{c,R}$ = Midspan delfection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,\,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

<u>PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)</u>

$$\Delta_{cx} = 8.03 + 3.20 + 0.84 = 12.07 \text{ mm}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels ($D+LL_{sus}$ *and* $D+LL_{full}$).

Since in this example the panel is squared, $\Delta_{cx} = \Delta_{cy} = 12.07$ mm and $\Delta_{mx} = \Delta_{my} = 6.63$ mm

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)}{2} = \left(\Delta_{cx} + \Delta_{my}\right) = \left(\Delta_{cy} + \Delta_{mx}\right) = 12.07 + 6.63 = 18.70 \text{ mm}$$

Table 7 – Immediate (Instantaneous) Deflections in the x-direction

Column Strip

Span	LDF	D						
		$\Delta_{frame-fixed}$, mm	$\Delta_{c-fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.756	5.21	8.03	0.00131	0.00035	3.20	0.84	12.07
Int	0.713	3.00	4.28	0.00035	0.00035	0.48	0.48	3.33

Span	LDF	D+LL$_{sus}$						
		$\Delta_{frame-fixed}$, mm	$\Delta_{c-fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.756	5.21	8.03	0.00131	0.00035	3.20	0.84	12.07
Int	0.713	3.00	4.28	0.00035	0.00035	0.48	0.48	3.33

Span	LDF	D+LL$_{full}$						
		$\Delta_{frame-fixed}$, mm	$\Delta_{c-fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.756	10.54	15.94	0.00184	0.00049	6.27	1.66	23.87
Int	0.713	5.71	8.14	0.00049	0.00049	0.90	0.90	6.34

Span	LDF	LL
		Δ_{cx}, mm
Ext	0.756	11.80
Int	0.713	3.01

Middle Strip

LDF	D						
	$\Delta_{frame-fixed}$, mm	$\Delta_{m-fixed}$, mm	θ_{m1}, rad	θ_{m2}, mm	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.244	5.31	2.59	0.00131	0.00035	3.20	0.84	6.63
0.288	3.01	1.83	0.00035	0.00035	0.48	0.48	0.78

LDF	D+LL$_{sus}$						
	$\Delta_{frame-fixed}$, mm	$\Delta_{m-fixed}$, mm	θ_{m1}, mm	θ_{m2}, mm	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.244	5.31	2.59	0.00131	0.00035	3.20	0.84	6.63
0.288	3.01	1.83	0.00035	0.00035	0.48	0.48	0.78

LDF	D+LL$_{full}$						
	$\Delta_{frame-fixed}$, mm	$\Delta_{m-fixed}$, mm	θ_{m1}, mm	θ_{m2}, mm	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.244	10.54	5.14	0.00184	0.00049	6.27	1.66	13.07
0.288	5.71	3.29	0.00049	0.00049	0.90	0.90	1.48

LDF	LL
	Δ_{mx}, mm
0.244	6.44
0.288	0.70

5.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst} =$ Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50\rho'}$$ ***ACI 318-14 (24.2.4.1.1)***

$(\Delta_{total})_{lt} =$ Time-dependent (long-term) total delfection, in.

$(\Delta_{total})_{Inst} =$ Total immediate (instantaneous) deflection, in.

For the exterior span

$\xi = 2$, consider the sustained load duration to be 60 months or more. ***ACI 318-14 (Table 24.2.4.1.3)***

$\rho' = 0$, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 12.07 = 24.14 \text{ mm}$$

$$(\Delta_{total})_{lt} = 12.07\times(1+2)+(23.87-12.07) = 48.01 \text{ mm}$$

Table 8 shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

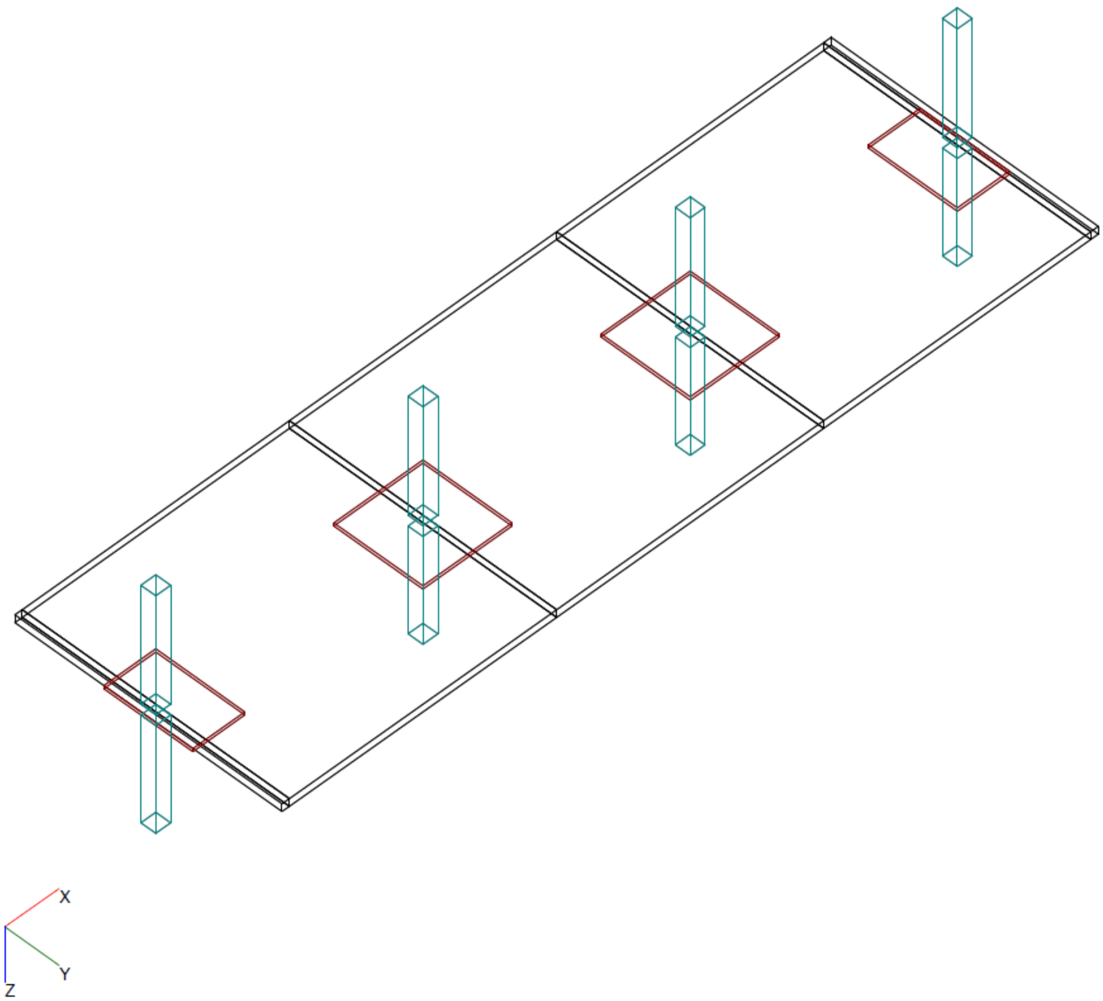
Table 8 - Long-Term Deflections

Span	$(\Delta_{sust})_{Inst}$, mm	λ_{Δ}	Δ_{cs}, mm	$(\Delta_{total})_{Inst}$, mm	$(\Delta_{total})_{lt}$, mm
Column Strip					
Exterior	12.07	2.000	24.14	23.87	48.01
Interior	3.33	2.000	6.66	6.34	13.00
Middle Strip					
Exterior	6.63	2.000	13.26	13.07	26.33
Interior	0.78	2.000	1.56	1.48	3.04

6. spSlab Software Program Model Solution

spSlab program utilizes the Elastic Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept ***(CSA A23.3-14 (13.8.2.6))***.

spSlab Program models the elastic frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips. The graphical and text results will be provided from the spSlab model in a future revision to this document.



spSlab v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

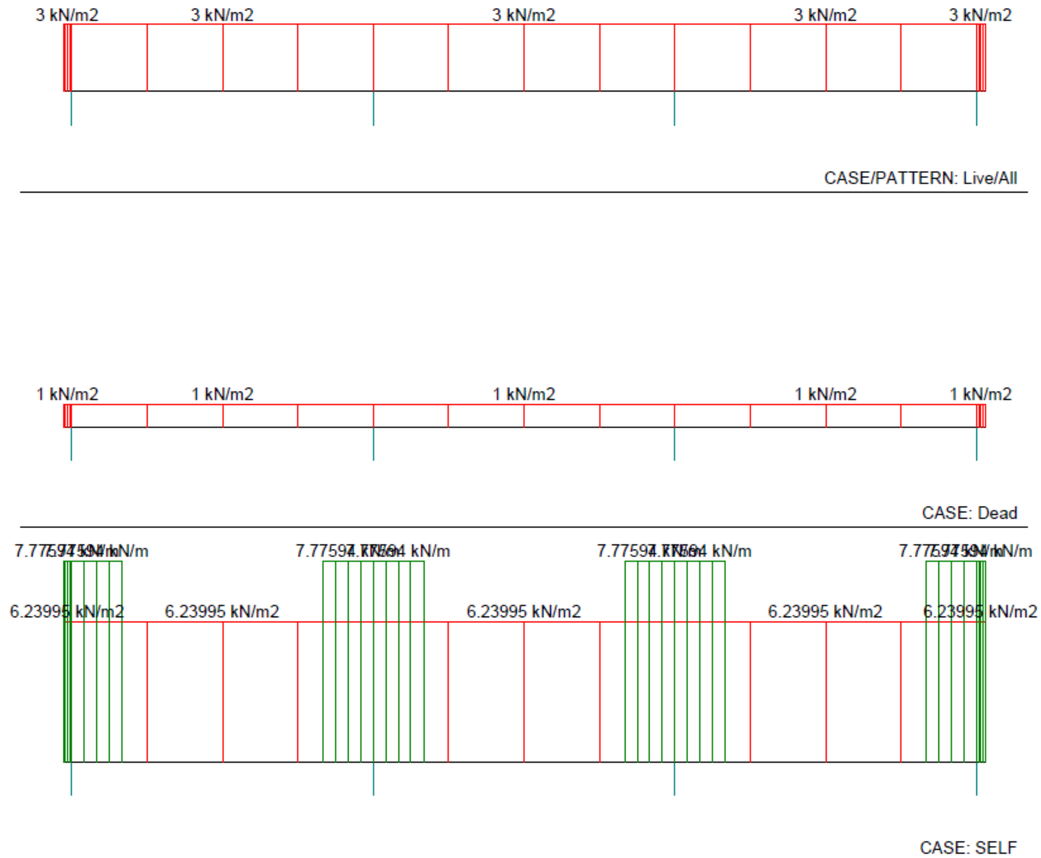
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 07/05/18

Time: 12:10:12



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File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

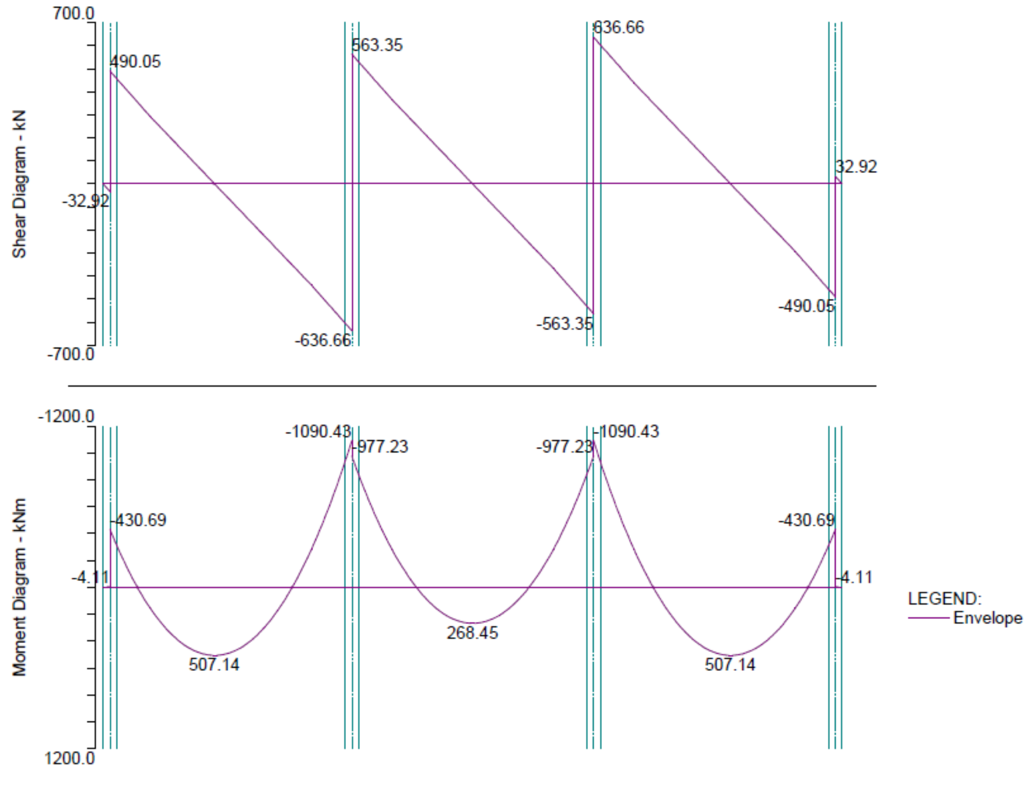
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 07/05/18

Time: 12:16:37



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File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

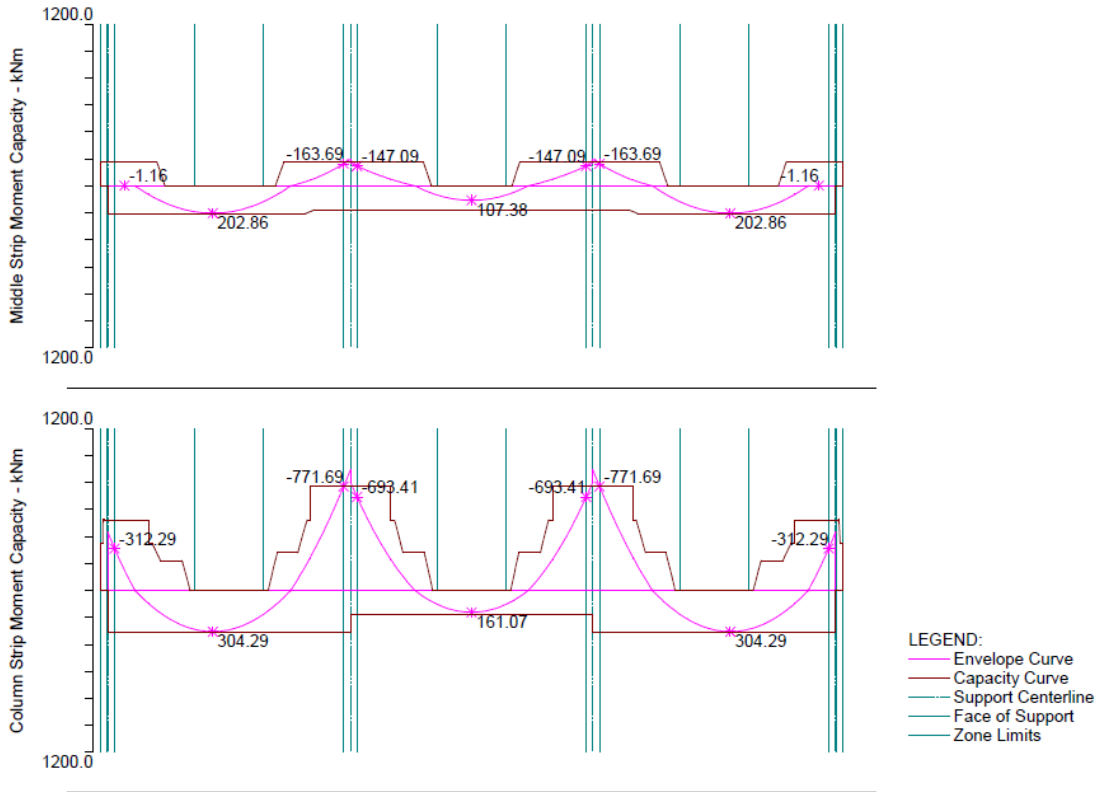
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 07/05/18

Time: 12:16:59



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File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

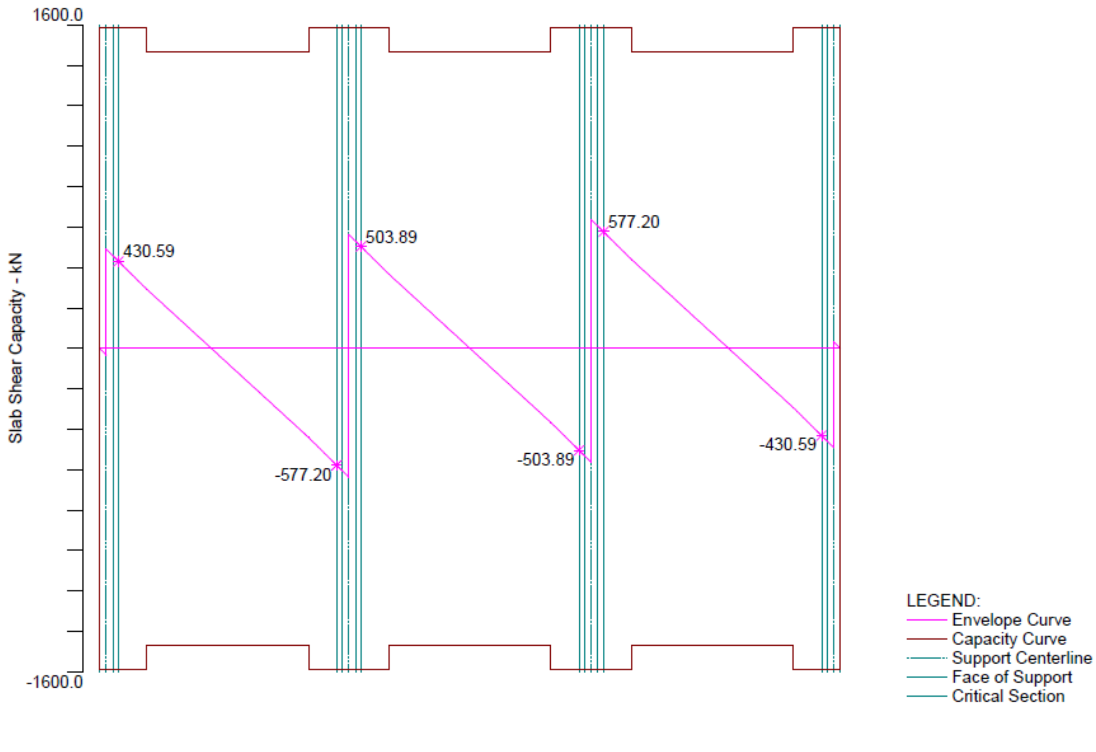
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 07/05/18

Time: 12:17:14



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File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

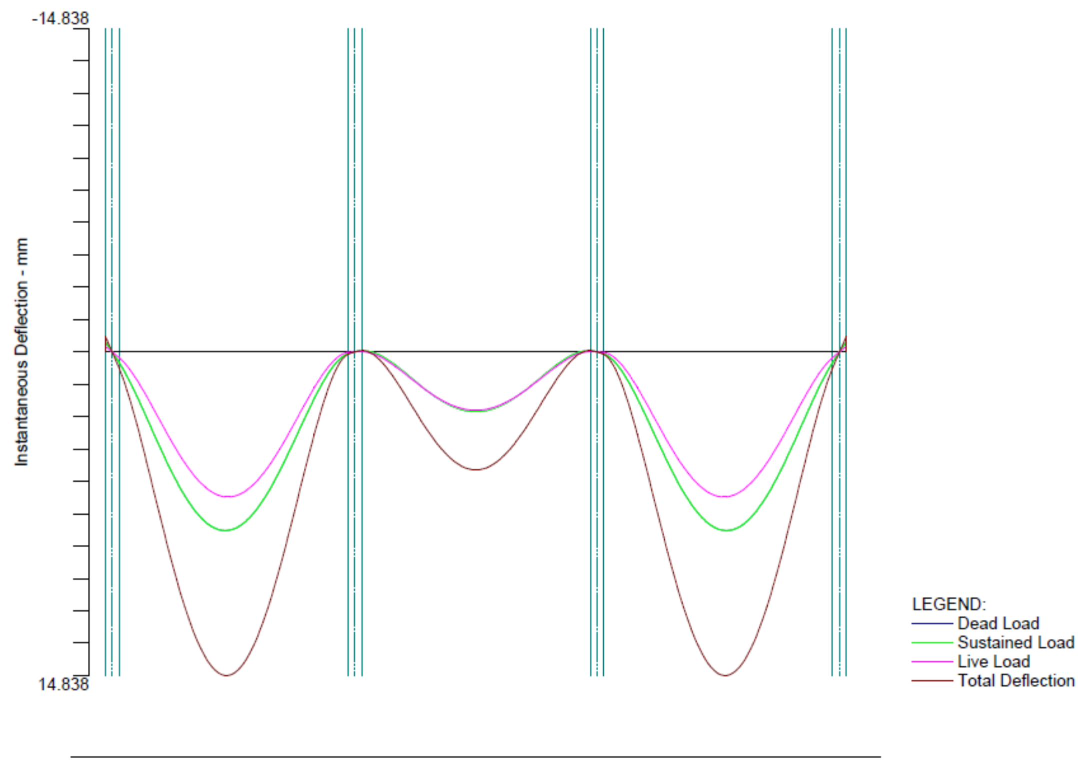
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 08/06/18

Time: 15:57:02



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File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

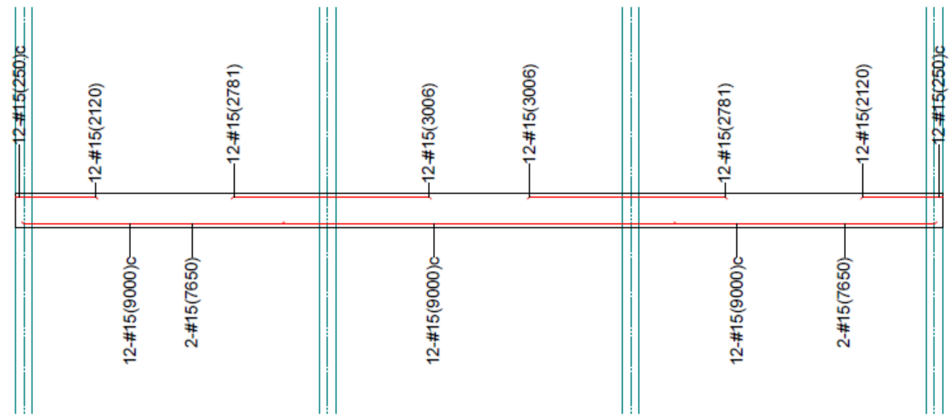
Frame: Interior Frame

Engineer: SP

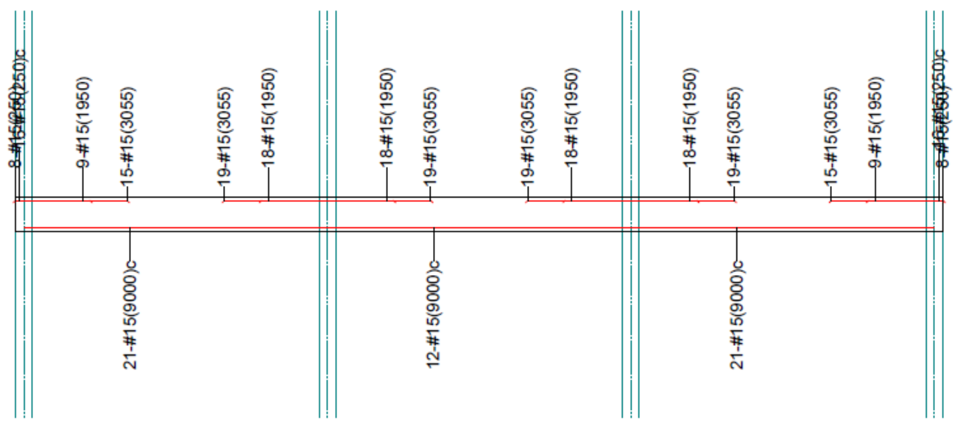
Code: CSA A23.3-14

Date: 07/05/18

Time: 12:17:46



Middle Strip Flexural Reinforcement



Column Strip Flexural Reinforcement

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File: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb

Project: Two Way Slab with Drop Panels

Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 07/05/18

Time: 12:18:30

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C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb Page 1

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 ooooo    oo         oooooo     ooo     ooooo o   ooooo   (TM)
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[1] INPUT ECHO

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General Information

File name: C:\TSDA\Two-Way Flate Slab Floor with Drop Panel (CSA)\Interior Strip.slb
Project: Two Way Slab with Drop Panels
Frame: Interior Frame
Engineer: SP
Code: CSA A23.3-14
Reinforcement Database: CSA G30.18
Mode: Design
Number of supports = 4 + Left cantilever + Right cantilever
Floor System: Two-Way

Live load pattern ratio = 0%
Minimum free edge distance for punching shear = 4 times slab effective depth.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Combined M-V-T reinforcement design NOT selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

Material Properties

		Slabs|Beams	Columns	
wc	=	2447.3	2447.3	kg/m3
f'c	=	35	42	MPa
Ec	=	29002	31047	MPa
fr	=	1.7748	3.8884	MPa

Precast concrete construction is not selected.

fy	=	400	MPa, Bars are not epoxy-coated
fyt	=	400	MPa
Es	=	199950	MPa

Reinforcement Database

Units: Db (mm), Ab (mm^2), Wb (kg/m)

Size	Db	Ab	Wb	Size	Db	Ab	Wb
#10	11	100	1	#15	16	200	2
#20	20	300	2	#25	25	500	4

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#30	30	700	5	#35	36	1000	8
#45	44	1500	12	#55	56	2500	20

Span Data

Slabs

Units: L1, wL, wR, L2L, L2R (m); t, Hmin (mm)

Span	Loc	L1	t	wL	wR	L2L	L2R	Hmin	
1	Int	0.250	260	4.500	4.500	9.000	9.000	---	LC *i
2	Int	9.000	260	4.500	4.500	9.000	9.000	252	
3	Int	9.000	260	4.500	4.500	9.000	9.000	252	
4	Int	9.000	260	4.500	4.500	9.000	9.000	252	
5	Int	0.250	260	4.500	4.500	9.000	9.000	---	RC *i

NOTES:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Support Data

Columns

Units: c1a, c2a, c1b, c2b (mm); Ha, Hb (m)

Supp	c1a	c2a	Ha	c1b	c2b	Hb	Red%
1	500	500	4.000	500	500	4.000	100
2	500	500	4.000	500	500	4.000	100
3	500	500	4.000	500	500	4.000	100
4	500	500	4.000	500	500	4.000	100

Drop Panels

Units: h (mm); Ll, Lr, Wl, Wr (m)

Supp	h	Ll	Lr	Wl	Wr	
1	108	0.250	1.500	1.500	1.500	*b
2	108	1.500	1.500	1.500	1.500	*b
3	108	1.500	1.500	1.500	1.500	*b
4	108	1.500	0.250	1.500	1.500	*b

*b - Standard drop.

Boundary Conditions

Units: Kz (kN/mm); Kry (kN-mm/rad)

Supp	Spring Kz	Spring Kry	Far End A	Far End B
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

Load Data

Load Cases and Combinations

Case	SELF	Dead	Live
Type	DEAD	DEAD	LIVE
U1	1.250	1.250	1.500

Area Loads

Units: Wa (kN/m2)

Case/Patt	Span	Wa
SELF	1	6.24
	2	6.24
	3	6.24
	4	6.24
	5	6.24
Dead	1	1.00
	2	1.00
	3	1.00
	4	1.00
	5	1.00
Live	1	3.00
	2	3.00
	3	3.00
	4	3.00
	5	3.00

Line Loads

Units: Wa, Wb (kN/m), La, Lb (m)

Case/Patt	Span	Wa	La	Wb	Lb

```
   SELF          1         7.78        0.000         7.78        0.250
                 2         7.78        0.000         7.78        1.500
                 2         7.78        7.500         7.78        9.000
                 3         7.78        0.000         7.78        1.500
                 3         7.78        7.500         7.78        9.000
                 4         7.78        0.000         7.78        1.500
                 4         7.78        7.500         7.78        9.000
                 5         7.78        0.000         7.78        0.250

Reinforcement Criteria
======================
   Slabs and Ribs
   --------------
                  _____Top bars___   __Bottom bars___
                     Min      Max      Min      Max
                  -------  -------  -------  -------
     Bar Size        #15      #15      #15      #15
     Bar spacing      25      457       25      457 mm
     Reinf ratio    0.14     5.00     0.14     5.00 %
     Cover            28                28          mm
     There is NOT more than 300 mm of concrete below top bars.

   Beams
   -----
                  _____Top bars___   __Bottom bars___    ____Stirrups____
                     Min      Max      Min      Max        Min      Max
                  -------  -------  -------  -------    -------  -------
     Bar Size        #20      #35      #20      #35        #10      #20
     Bar spacing      25      457       25      457        152      457 mm
     Reinf ratio    0.14     5.00     0.14     5.00 %
     Cover            38                38          mm
     Layer dist.      25                25          mm
     No. of legs                                             2        6
     Side cover                                             38          mm
     1st Stirrup                                            76          mm
     There is NOT more than 300 mm of concrete below top bars.
```

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 ooooo    oo         oooooo     ooo      ooooo o   ooooo   (TM)
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Reinforced Concrete Beams, One-way and Two-way Slab Systems

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[2] DESIGN RESULTS*

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*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

Strip Widths and Distribution Factors

Units: Width (m).

Span	Strip	Width Left**	Width Right**	Width Bottom*	Moment Factor Left**	Moment Factor Right**	Moment Factor Bottom*
1	Column	4.50	4.50	4.50	1.000	1.000	0.600
	Middle	4.50	4.50	4.50	0.000	0.000	0.400
2	Column	4.50	4.50	4.50	1.000	0.825	0.600
	Middle	4.50	4.50	4.50	0.000	0.175	0.400
3	Column	4.50	4.50	4.50	0.825	0.825	0.600
	Middle	4.50	4.50	4.50	0.175	0.175	0.400
4	Column	4.50	4.50	4.50	0.825	1.000	0.600
	Middle	4.50	4.50	4.50	0.175	0.000	0.400
5	Column	4.50	4.50	4.50	1.000	1.000	0.600
	Middle	4.50	4.50	4.50	0.000	0.000	0.400

*Used for bottom reinforcement. **Used for top reinforcement.

Top Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Strip	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	Left	4.50	0.38	0.072	2340	30207	5	281	16-#15 *3
		Midspan	4.50	1.22	0.134	2340	44772	11	281	16-#15 *3
		Right	4.50	2.85	0.206	2988	29848	25	188	24-#15 *3 *5
	Middle	Left	4.50	0.00	0.000	2340	30207	0	375	12-#15 *3
		Midspan	4.50	0.00	0.103	2340	30207	0	375	12-#15 *3
		Right	4.50	0.00	0.206	2340	30207	0	375	12-#15 *3
2	Column	Left	4.50	312.29	0.250	2988	29848	2843	188	24-#15 *3
		Midspan	4.50	0.00	4.500	0	30207	0	0	---
		Right	4.50	771.69	8.750	2988	29848	7344	122	37-#15
	Middle	Left	4.50	1.16	0.618	2340	30207	15	375	12-#15 *3
		Midspan	4.50	0.00	4.500	0	30207	0	0	---
		Right	4.50	163.69	8.750	2340	30207	2194	375	12-#15 *3
3	Column	Left	4.50	693.41	0.250	2988	29848	6546	122	37-#15
		Midspan	4.50	0.00	4.500	0	30207	0	0	---
		Right	4.50	693.41	8.750	2988	29848	6546	122	37-#15
	Middle	Left	4.50	147.09	0.250	2340	30207	1967	375	12-#15 *3

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		Midspan	4.50	0.00	4.500	0	30207	0	0	---
		Right	4.50	147.09	8.750	2340	30207	1967	375	12-#15 *3
4	Column	Left	4.50	771.69	0.250	2988	29848	7344	122	37-#15
		Midspan	4.50	0.00	4.500	0	30207	0	0	---
		Right	4.50	312.29	8.750	2988	29848	2843	188	24-#15 *3
	Middle	Left	4.50	163.69	0.250	2340	30207	2194	375	12-#15 *3
		Midspan	4.50	0.00	4.500	0	30207	0	0	---
		Right	4.50	1.16	8.382	2340	30207	15	375	12-#15 *3
5	Column	Left	4.50	2.85	0.044	2988	29848	25	188	24-#15 *3 *5
		Midspan	4.50	1.22	0.116	2340	44772	11	281	16-#15 *3
		Right	4.50	0.38	0.178	2340	30207	5	281	16-#15 *3
	Middle	Left	4.50	0.00	0.044	2340	30207	0	375	12-#15 *3
		Midspan	4.50	0.00	0.147	2340	30207	0	375	12-#15 *3
		Right	4.50	0.00	0.250	2340	30207	0	375	12-#15 *3

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Top Bar Details

Units: Length (m)

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	Column	---		---		16-#15	0.25	8-#15	0.25	---	
	Middle	---		---		12-#15	0.25	---		---	
2	Column	15-#15	3.06	9-#15	1.95	---		19-#15	3.06	18-#15*	1.95
	Middle	12-#15	2.12	---		---		12-#15	2.78	---	
3	Column	19-#15	3.06	18-#15*	1.95	---		19-#15	3.06	18-#15*	1.95
	Middle	12-#15	3.01	---		---		12-#15	3.01	---	
4	Column	19-#15	3.06	18-#15*	1.95	---		15-#15	3.06	9-#15	1.95
	Middle	12-#15	2.78	---		---		12-#15	2.12	---	
5	Column	8-#15	0.25	---		16-#15	0.25	---		---	
	Middle	---		---		12-#15	0.25	---		---	

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Top Bar Development Lengths

Units: Length (mm)

		Left				Continuous		Right			
Span	Strip	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
1	Column	---		---		16-#15	300.00	8-#15	300.00	---	
	Middle	---		---		12-#15	300.00	---		---	
2	Column	15-#15	300.00	9-#15	300.00	---		19-#15	344.77	18-#15	344.77
	Middle	12-#15	300.00	---		---		12-#15	317.60	---	
3	Column	19-#15	307.32	18-#15	307.32	---		19-#15	307.32	18-#15	307.32
	Middle	12-#15	300.00	---		---		12-#15	300.00	---	
4	Column	19-#15	344.77	18-#15	344.77	---		15-#15	300.00	9-#15	300.00
	Middle	12-#15	317.60	---		---		12-#15	300.00	---	
5	Column	8-#15	300.00	---		16-#15	300.00	---		---	
	Middle	---		---		12-#15	300.00	---		---	

Band Reinforcement at Supports

Units: Width (mm), As (mm^2)

Supp	Width<C>	Width<B>	Width<S>	As<C>	As<B>	As<S>	Bars<C>	Bars<B>	Bars<S>
1	4500	1604	2896	4800	3000	1800	24-#15	15-#15	9-#15
2	4500	1604	2896	7400	3000	4400	37-#15	15-#15	22-#15
3	4500	1604	2896	7400	3000	4400	37-#15	15-#15	22-#15
4	4500	1604	2896	4800	3000	1800	24-#15	15-#15	9-#15

<C> Total Strip, <B> Banded Strip, <S> Remaining Strip

Bottom Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Strip	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	4.50	0.00	0.103	0	30207	0	0	---
	Middle	4.50	0.00	0.103	0	30207	0	0	---
2	Column	4.50	304.29	3.900	2340	30207	4156	214	21-#15
	Middle	4.50	202.86	3.900	2340	30207	2733	321	14-#15

Span	Strip									
3	Column	4.50	161.07	4.500	2340	30207	2158	375	12-#15	*3
	Middle	4.50	107.38	4.500	2340	30207	1429	375	12-#15	*3
4	Column	4.50	304.29	5.100	2340	30207	4156	214	21-#15	
	Middle	4.50	202.86	5.100	2340	30207	2733	321	14-#15	
5	Column	4.50	0.00	0.147	0	30207	0	0	---	
	Middle	4.50	0.00	0.147	0	30207	0	0	---	

NOTES:
*3 - Design governed by minimum reinforcement.

Bottom Bar Details

Units: Start (m), Length (m)

Span	Strip	Long Bars			Short Bars		
		Bars	Start	Length	Bars	Start	Length
1	Column	---			---		
	Middle	---			---		
2	Column	21-#15	0.00	9.00	---		
	Middle	12-#15	0.00	9.00	2-#15	0.00	7.65
3	Column	12-#15	0.00	9.00	---		
	Middle	12-#15	0.00	9.00	---		
4	Column	21-#15	0.00	9.00	---		
	Middle	12-#15	0.00	9.00	2-#15	1.35	7.65
5	Column	---			---		
	Middle	---			---		

Bottom Bar Development Lengths

Units: DevLen (mm)

Span	Strip	Long Bars		Short Bars	
		Bars	DevLen	Bars	DevLen
1	Column	---		---	
	Middle	---		---	
2	Column	21-#15	343.76	---	
	Middle	12-#15	339.10	2-#15	339.10
3	Column	12-#15	312.40	---	
	Middle	12-#15	300.00	---	
4	Column	21-#15	343.76	---	
	Middle	12-#15	339.10	2-#15	339.10
5	Column	---		---	
	Middle	---		---	

Flexural Capacity

Units: x (m), As (mm^2), PhiMn, Mu (kNm)

Span	Strip	x	Top						Bottom					
			AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	Column	0.000	4800	-349.26	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.072	4800	-525.51	-0.38	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.125	4800	-525.51	-1.03	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.134	4800	-517.36	-1.22	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.206	4800	-517.36	-2.85	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.250	4800	-517.36	-4.11	U1	All	---	0	0.00	0.00	U1	All	---
	Middle	0.000	2400	-178.71	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.072	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.125	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.134	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.206	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.250	2400	-178.71	-0.00	U1	All	---	0	0.00	0.00	U1	All	---
2	Column	0.000	4800	-517.36	-432.91	U1	All	---	4200	307.38	0.00	U1	All	---
		0.250	4800	-517.36	-312.29	U1	All	OK	4200	307.38	0.00	U1	All	OK
		1.500	4800	-517.36	0.00	U1	All	OK	4200	307.38	93.70	U1	All	OK
		1.500	4800	-349.26	0.00	U1	All	OK	4200	307.38	93.80	U1	All	OK
		1.650	4800	-349.26	0.00	U1	All	OK	4200	307.38	119.26	U1	All	OK
		1.950	3000	-222.11	0.00	U1	All	OK	4200	307.38	165.33	U1	All	OK
		2.755	3000	-222.11	0.00	U1	All	OK	4200	307.38	256.38	U1	All	OK
		3.055	0	0.00	0.00	U1	All	OK	4200	307.38	278.19	U1	All	OK
		3.225	0	0.00	0.00	U1	All	OK	4200	307.38	287.67	U1	All	OK
		3.900	0	0.00	0.00	U1	All	OK	4200	307.38	304.29	U1	All	OK
		4.500	0	0.00	0.00	U1	All	OK	4200	307.38	291.07	U1	All	OK
		5.775	0	0.00	0.00	U1	All	OK	4200	307.38	175.52	U1	All	OK
		5.945	0	0.00	0.00	U1	All	OK	4200	307.38	151.08	U1	All	OK
		6.290	3800	-279.19	0.00	U1	All	OK	4200	307.38	95.13	U1	All	OK

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	7.050	3800	-279.19	-84.54	U1	All	OK	4200	307.38	0.00	U1	All	OK
	7.395	7400	-524.82	-203.09	U1	All	OK	4200	307.38	0.00	U1	All	OK
	7.500	7400	-524.82	-241.14	U1	All	OK	4200	307.38	0.00	U1	All	OK
	7.500	7400	-777.16	-241.36	U1	All	OK	4200	307.38	0.00	U1	All	OK
	8.750	7400	-777.16	-771.69	U1	All	OK	4200	307.38	0.00	U1	All	OK
	8.813	7400	-777.16	-801.78	U1	All	---	4200	307.38	0.00	U1	All	---
	9.000	7400	-777.16	-893.99	U1	All	---	4200	307.38	0.00	U1	All	---
Middle	0.000	2400	-178.71	2.22	U1	All	---	2800	207.70	0.00	U1	All	---
	0.250	2400	-178.71	-0.00	U1	All	OK	2800	207.70	0.00	U1	All	OK
	0.618	2400	-178.71	-1.16	U1	All	OK	2800	207.70	0.00	U1	All	OK
	1.820	2400	-178.71	0.00	U1	All	OK	2800	207.70	97.42	U1	All	OK
	2.120	0	0.00	0.00	U1	All	OK	2800	207.70	125.65	U1	All	OK
	3.225	0	0.00	0.00	U1	All	OK	2800	207.70	191.78	U1	All	OK
	3.900	0	0.00	0.00	U1	All	OK	2800	207.70	202.86	U1	All	OK
	4.500	0	0.00	0.00	U1	All	OK	2800	207.70	194.05	U1	All	OK
	5.775	0	0.00	0.00	U1	All	OK	2800	207.70	117.01	U1	All	OK
	6.219	0	0.00	0.00	U1	All	OK	2800	207.70	71.59	U1	All	OK
	6.536	2400	-178.71	0.00	U1	All	OK	2800	207.70	33.18	U1	All	OK
	7.311	2400	-178.71	-29.48	U1	All	OK	2800	207.70	0.00	U1	All	OK
	7.650	2400	-178.71	-53.47	U1	All	OK	2400	178.71	0.00	U1	All	OK
	8.750	2400	-178.71	-163.69	U1	All	OK	2400	178.71	0.00	U1	All	OK
	8.875	2400	-178.71	-179.68	U1	All	---	2400	178.71	0.00	U1	All	---
	9.000	2400	-178.71	-196.44	U1	All	---	2400	178.71	0.00	U1	All	---
3 Column	0.000	7400	-777.16	-806.21	U1	All	---	2400	178.71	0.00	U1	All	---
	0.063	7400	-777.16	-777.38	U1	All	---	2400	178.71	0.00	U1	All	---
	0.250	7400	-777.16	-693.41	U1	All	OK	2400	178.71	0.00	U1	All	OK
	1.500	7400	-777.16	-231.36	U1	All	OK	2400	178.71	0.00	U1	All	OK
	1.500	7400	-524.82	-231.18	U1	All	OK	2400	178.71	0.00	U1	All	OK
	1.643	7400	-524.82	-189.25	U1	All	OK	2400	178.71	0.00	U1	All	OK
	1.950	3800	-279.19	-105.63	U1	All	OK	2400	178.71	0.00	U1	All	OK
	2.748	3800	-279.19	0.00	U1	All	OK	2400	178.71	48.68	U1	All	OK
	3.055	0	0.00	0.00	U1	All	OK	2400	178.71	84.64	U1	All	OK
	3.225	0	0.00	0.00	U1	All	OK	2400	178.71	101.60	U1	All	OK
	4.500	0	0.00	0.00	U1	All	OK	2400	178.71	161.07	U1	All	OK
	5.775	0	0.00	0.00	U1	All	OK	2400	178.71	101.60	U1	All	OK
	5.945	0	0.00	0.00	U1	All	OK	2400	178.71	84.64	U1	All	OK
	6.252	3800	-279.19	0.00	U1	All	OK	2400	178.71	48.68	U1	All	OK
	7.050	3800	-279.19	-105.63	U1	All	OK	2400	178.71	0.00	U1	All	OK
	7.357	7400	-524.82	-189.25	U1	All	OK	2400	178.71	0.00	U1	All	OK
	7.500	7400	-524.82	-231.18	U1	All	OK	2400	178.71	0.00	U1	All	OK
	7.500	7400	-777.16	-231.36	U1	All	OK	2400	178.71	0.00	U1	All	OK
	8.750	7400	-777.16	-693.41	U1	All	OK	2400	178.71	0.00	U1	All	OK
	8.938	7400	-777.16	-777.38	U1	All	---	2400	178.71	0.00	U1	All	---
	9.000	7400	-777.16	-806.21	U1	All	---	2400	178.71	0.00	U1	All	---
Middle	0.000	2400	-178.71	-171.01	U1	All	---	2400	178.71	0.00	U1	All	---
	0.250	2400	-178.71	-147.09	U1	All	OK	2400	178.71	0.00	U1	All	OK
	2.706	2400	-178.71	0.00	U1	All	OK	2400	178.71	28.89	U1	All	OK
	3.006	0	0.00	0.00	U1	All	OK	2400	178.71	52.95	U1	All	OK
	3.225	0	0.00	0.00	U1	All	OK	2400	178.71	67.73	U1	All	OK
	4.500	0	0.00	0.00	U1	All	OK	2400	178.71	107.38	U1	All	OK
	5.775	0	0.00	0.00	U1	All	OK	2400	178.71	67.73	U1	All	OK
	5.994	0	0.00	0.00	U1	All	OK	2400	178.71	52.95	U1	All	OK
	6.294	2400	-178.71	0.00	U1	All	OK	2400	178.71	28.89	U1	All	OK
	8.750	2400	-178.71	-147.09	U1	All	OK	2400	178.71	0.00	U1	All	OK
	9.000	2400	-178.71	-171.01	U1	All	---	2400	178.71	0.00	U1	All	---
4 Column	0.000	7400	-777.16	-893.99	U1	All	---	4200	307.38	0.00	U1	All	---
	0.188	7400	-777.16	-801.78	U1	All	---	4200	307.38	0.00	U1	All	---
	0.250	7400	-777.16	-771.69	U1	All	OK	4200	307.38	0.00	U1	All	OK
	1.500	7400	-777.16	-241.36	U1	All	OK	4200	307.38	0.00	U1	All	OK
	1.500	7400	-524.82	-241.14	U1	All	OK	4200	307.38	0.00	U1	All	OK
	1.605	7400	-524.82	-203.09	U1	All	OK	4200	307.38	0.00	U1	All	OK
	1.950	3800	-279.19	-84.54	U1	All	OK	4200	307.38	0.00	U1	All	OK
	2.710	3800	-279.19	0.00	U1	All	OK	4200	307.38	95.13	U1	All	OK
	3.055	0	0.00	0.00	U1	All	OK	4200	307.38	151.08	U1	All	OK
	3.225	0	0.00	0.00	U1	All	OK	4200	307.38	175.52	U1	All	OK
	4.500	0	0.00	0.00	U1	All	OK	4200	307.38	291.07	U1	All	OK
	5.100	0	0.00	0.00	U1	All	OK	4200	307.38	304.29	U1	All	OK
	5.775	0	0.00	0.00	U1	All	OK	4200	307.38	287.67	U1	All	OK
	5.945	0	0.00	0.00	U1	All	OK	4200	307.38	278.19	U1	All	OK
	6.245	3000	-222.11	0.00	U1	All	OK	4200	307.38	256.38	U1	All	OK
	7.050	3000	-222.11	0.00	U1	All	OK	4200	307.38	165.33	U1	All	OK
	7.350	4800	-349.26	0.00	U1	All	OK	4200	307.38	119.26	U1	All	OK
	7.500	4800	-349.26	0.00	U1	All	OK	4200	307.38	93.80	U1	All	OK
	7.500	4800	-517.36	0.00	U1	All	OK	4200	307.38	93.70	U1	All	OK
	8.750	4800	-517.36	-312.29	U1	All	OK	4200	307.38	0.00	U1	All	OK
	9.000	4800	-517.36	-432.91	U1	All	---	4200	307.38	0.00	U1	All	---
Middle	0.000	2400	-178.71	-196.44	U1	All	---	2400	178.71	0.00	U1	All	---
	0.125	2400	-178.71	-179.68	U1	All	---	2400	178.71	0.00	U1	All	---
	0.250	2400	-178.71	-163.69	U1	All	OK	2400	178.71	0.00	U1	All	OK
	1.350	2400	-178.71	-53.47	U1	All	OK	2400	178.71	0.00	U1	All	OK
	1.689	2400	-178.71	-29.48	U1	All	OK	2800	207.70	0.00	U1	All	OK
	2.464	2400	-178.71	0.00	U1	All	OK	2800	207.70	33.18	U1	All	OK
	2.781	0	0.00	0.00	U1	All	OK	2800	207.70	71.59	U1	All	OK
	3.225	0	0.00	0.00	U1	All	OK	2800	207.70	117.01	U1	All	OK
	4.500	0	0.00	0.00	U1	All	OK	2800	207.70	194.05	U1	All	OK

	5.100	0	0.00	0.00	U1	All	OK	2800	207.70	202.86	U1	All	OK
	5.775	0	0.00	0.00	U1	All	OK	2800	207.70	191.78	U1	All	OK
	6.880	0	0.00	0.00	U1	All	OK	2800	207.70	125.65	U1	All	OK
	7.180	2400	-178.71	0.00	U1	All	OK	2800	207.70	97.42	U1	All	OK
	8.382	2400	-178.71	-1.16	U1	All	OK	2800	207.70	0.00	U1	All	OK
	8.750	2400	-178.71	-0.00	U1	All	OK	2800	207.70	0.00	U1	All	OK
	9.000	2400	-178.71	2.22	U1	All	---	2800	207.70	0.00	U1	All	---
5 Column	0.000	4800	-517.36	-4.11	U1	All	---	0	0.00	0.00	U1	All	---
	0.044	4800	-517.36	-2.85	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.116	4800	-525.51	-1.22	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.125	4800	-525.51	-1.03	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.178	4800	-349.26	-0.38	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.250	4800	-349.26	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
Middle	0.000	2400	-178.71	-0.00	U1	All	---	0	0.00	0.00	U1	All	---
	0.044	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.116	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.125	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.178	2400	-178.71	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.250	2400	-178.71	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK

Slab Shear Capacity

Units: b, dv (mm), Xu (m), PhiVc, Vu(kN)

Span	b	dv	Beta	Vratio	PhiVc	Vu	Xu
1	9000	202	0.000	1.000	1465.21	0.00	0.00
	9000	234	0.000	1.000	1587.23	0.00	0.00
2	9000	234	0.000	1.000	1587.23	430.59	0.45
	9000	202	0.000	1.000	1465.21	439.15	7.50
	9000	234	0.000	1.000	1587.23	577.20	8.55
3	9000	234	0.000	1.000	1587.23	503.89	0.45
	9000	202	0.000	1.000	1465.21	365.85	7.50
	9000	234	0.000	1.000	1587.23	503.89	8.55
4	9000	234	0.000	1.000	1587.23	577.20	0.45
	9000	202	0.000	1.000	1465.21	439.15	1.50
	9000	234	0.000	1.000	1587.23	430.59	8.55
5	9000	234	0.000	1.000	1587.23	0.00	0.25
	9000	202	0.000	1.000	1465.21	0.00	0.25

Flexural Transfer of Negative Unbalanced Moment at Supports

Units: Width (mm), Munb (kNm), As (mm^2)

Supp	Width	Width-c	d	Munb	Comb	Pat	GammaF	AsReq	AsProv	Add Bars
1	1604	1604	332	426.58	U1	All	0.626	2475	3000	---
2	1604	1604	332	113.21	U1	All	0.600	608	3000	---
3	1604	1604	332	113.21	U1	All	0.600	608	3000	---
4	1604	1604	332	426.58	U1	All	0.626	2475	3000	---

Punching Shear Around Columns

Critical Section Properties

Units: b1, b2, b0, davg, CG, c(left), c(right) (mm), Ac (mm^2), Jc (mm^4)

Supp	Type	b1	b2	b0	davg	CG	c(left)	c(right)	Ac	Jc
1	Rect	666.0	832.0	2164.0	332.0	211.0	461.0	205.0	7.1845e+005	3.9262e+010
2	Rect	832.0	832.0	3328.0	332.0	0.0	416.0	416.0	1.1049e+006	1.3255e+011
3	Rect	832.0	832.0	3328.0	332.0	0.0	416.0	416.0	1.1049e+006	1.3255e+011
4	Rect	666.0	832.0	2164.0	332.0	-211.0	205.0	461.0	7.1845e+005	3.9262e+010

Punching Shear Results

Units: Vu (kN), Munb (kNm), vu (N/mm^2), Phi*vc (N/mm^2)

Supp	Vu	vu	Munb	Comb	Pat	GammaV	vu	Phi*vc
1	515.46	0.717	317.80	U1	All	0.374	1.337	1.426
2	1190.63	1.078	-113.21	U1	All	0.400	1.220	1.426
3	1190.63	1.078	113.21	U1	All	0.400	1.220	1.426
4	515.46	0.717	-317.80	U1	All	0.374	1.337	1.426

Punching Shear Around Drops

Critical Section Properties

Units: b1, b2, b0, davg, CG, c(left), c(right) (mm), Ac (mm^2), Jc (mm^4)

Supp	Type	b1	b2	b0	davg	CG	c(left)	c(right)	Ac	Jc
1	Rect	1862.0	3224.0	6948.0	224.0	1113.0	1363.0	499.0	1.5564e+006	5.8e+011
2	Rect	3224.0	3224.0	12896.0	224.0	0.0	1612.0	1612.0	2.8887e+006	5.0103e+012
3	Rect	3224.0	3224.0	12896.0	224.0	0.0	1612.0	1612.0	2.8887e+006	5.0103e+012
4	Rect	1862.0	3224.0	6948.0	224.0	-1113.0	499.0	1363.0	1.5564e+006	5.8e+011

Punching Shear Results

Units: Vu (kN), vu (N/mm^2), Phi*vc (N/mm^2)

Supp	Vu	Comb	Pat	vu	Phi*vc
1	441.62	U1	All	0.284	1.103
2	1059.17	U1	All	0.367	0.998
3	1059.17	U1	All	0.367	0.998
4	441.62	U1	All	0.284	1.103

Integrity Reinforcement at Supports

Units: Vse (kN), Asb (mm^2)

Supp	Vse	Asb
1	485.76	2429
2	1116.94	5585
3	1116.94	5585
4	485.76	2429

NOTES: The sum of bottom reinforcement crossing the perimeter of the support on all sides shall not be less than the above listed values.

Material Takeoff

Reinforcement in the Direction of Analysis

Top Bars:	1110.2 kg	<=>	40.37 kg/m	<=>	4.486 kg/m^2
Bottom Bars:	1319.7 kg	<=>	47.99 kg/m	<=>	5.332 kg/m^2
Stirrups:	0.0 kg	<=>	0.00 kg/m	<=>	0.000 kg/m^2
Total Steel:	2429.9 kg	<=>	88.36 kg/m	<=>	9.818 kg/m^2
Concrete:	67.4 m^3	<=>	2.45 m^3/m	<=>	0.272 m^3/m^2

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A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

[3] DEFLECTION RESULTS

Section Properties

Frame Section Properties

Units: Ig, Icr (mm^4), Mcr (kNm)

Span	Zone	M+ve Ig	M+ve Icr	M+ve Mcr	M-ve Ig	M-ve Icr	M-ve Mcr
1	Left	1.3182e+010	0.00000	179.96	1.3182e+010	1.4991e+009	-179.96
	Midspan	1.3182e+010	0.00000	179.96	1.3182e+010	1.8638e+009	-179.96
	Right	2.3132e+010	0.00000	190.40	2.3132e+010	3.6360e+009	-269.43
2	Left	2.3132e+010	1.4217e+009	190.40	2.3132e+010	3.6360e+009	-269.43
	Midspan	1.3182e+010	1.7292e+009	179.96	1.3182e+010	0.00000	-179.96
	Right	2.3132e+010	1.4217e+009	190.40	2.3132e+010	4.6354e+009	-269.43
3	Left	2.3132e+010	1.1061e+009	190.40	2.3132e+010	4.6354e+009	-269.43
	Midspan	1.3182e+010	1.3092e+009	179.96	1.3182e+010	0.00000	-179.96
	Right	2.3132e+010	1.1061e+009	190.40	2.3132e+010	4.6354e+009	-269.43
4	Left	2.3132e+010	1.4217e+009	190.40	2.3132e+010	4.6354e+009	-269.43
	Midspan	1.3182e+010	1.7292e+009	179.96	1.3182e+010	0.00000	-179.96
	Right	2.3132e+010	1.4217e+009	190.40	2.3132e+010	3.6360e+009	-269.43
5	Left	2.3132e+010	0.00000	190.40	2.3132e+010	3.6360e+009	-269.43
	Midspan	1.3182e+010	0.00000	179.96	1.3182e+010	1.8638e+009	-179.96
	Right	1.3182e+010	0.00000	179.96	1.3182e+010	1.4991e+009	-179.96

NOTES: M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (mm^4), Mmax (kNm)

			Load Level					
			Dead		Sustained		Dead+Live	
Span	Zone	Weight	Mmax	Ie	Mmax	Ie	Mmax	Ie
1	Right	1.000	-2.28	2.3132e+010	-2.28	2.3132e+010	-3.12	2.3132e+010
	Span Avg	----	----	2.3132e+010	----	2.3132e+010	----	2.3132e+010
2	Middle	0.500	271.75	5.0556e+009	271.75	5.0556e+009	383.39	2.9138e+009
	Right	0.500	-585.84	6.4346e+009	-585.84	6.4346e+009	-824.59	5.2806e+009
	Span Avg	----	----	5.7451e+009	----	5.7451e+009	----	4.0972e+009
3	Left	0.250	-525.03	7.1350e+009	-525.03	7.1350e+009	-738.99	5.5318e+009
	Middle	0.500	143.46	1.3182e+010	143.46	1.3182e+010	202.88	9.5968e+009
	Right	0.250	-525.03	7.1350e+009	-525.03	7.1350e+009	-738.99	5.5318e+009
	Span Avg	----	----	1.0158e+010	----	1.0158e+010	----	7.5643e+009
4	Left	0.500	-585.84	6.4346e+009	-585.84	6.4346e+009	-824.59	5.2806e+009
	Middle	0.500	271.75	5.0556e+009	271.75	5.0556e+009	383.39	2.9138e+009
	Span Avg	----	----	5.7451e+009	----	5.7451e+009	----	4.0972e+009
5	Left	1.000	-2.28	2.3132e+010	-2.28	2.3132e+010	-3.12	2.3132e+010
	Span Avg	----	----	2.3132e+010	----	2.3132e+010	----	2.3132e+010

Strip Section Properties at Midspan

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Units: Ig (mm^4)

Span	Column Strip Ig	Column Strip LDF	Column Strip Ratio	Middle Strip Ig	Middle Strip LDF	Middle Strip Ratio
1	6.591e+009	0.800	1.600	6.591e+009	0.200	0.400
2	6.591e+009	0.756	1.513	6.591e+009	0.244	0.488
3	6.591e+009	0.712	1.425	6.591e+009	0.288	0.575
4	6.591e+009	0.756	1.513	6.591e+009	0.244	0.488
5	6.591e+009	0.800	1.600	6.591e+009	0.200	0.400

NOTES: Load distirubtion factor, LDL, averages moment distribution factors listed in [2] Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.46	---	-0.25	-0.25	-0.46	-0.71
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	8.19	---	6.65	6.65	8.19	14.84
		Loc	4.200	---	4.275	4.275	4.200	4.275
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	2.73	---	2.67	2.67	2.73	5.41
		Loc	4.500	---	4.500	4.500	4.500	4.500
	Up	Def	-0.05	---	-0.01	-0.01	-0.05	-0.06
		Loc	0.324	---	0.250	0.250	0.324	0.324
4	Down	Def	8.19	---	6.65	6.65	8.19	14.84
		Loc	4.800	---	4.725	4.725	4.800	4.725
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.46	---	-0.25	-0.25	-0.46	-0.71
		Loc	0.250	---	0.250	0.250	0.250	0.250

Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.46	---	-0.25	-0.25	-0.46	-0.71
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	11.07	---	9.39	9.39	11.07	20.46
		Loc	4.275	---	4.350	4.350	4.275	4.350
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	4.15	---	3.88	3.88	4.15	8.03
		Loc	4.500	---	4.500	4.500	4.500	4.500
	Up	Def	-0.04	---	-0.01	-0.01	-0.04	-0.05
		Loc	0.250	---	0.188	0.188	0.250	0.250
4	Down	Def	11.07	---	9.39	9.39	11.07	20.46
		Loc	4.725	---	4.650	4.650	4.725	4.650
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.46	---	-0.25	-0.25	-0.46	-0.71
		Loc	0.250	---	0.250	0.250	0.250	0.250

Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.46	---	-0.25	-0.25	-0.46	-0.71
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	5.34	---	3.92	3.92	5.34	9.26
		Loc	3.975	---	4.125	4.125	3.975	4.050
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	1.32	---	1.47	1.47	1.32	2.78
		Loc	4.500	---	4.500	4.500	4.500	4.500
	Up	Def	-0.07	---	-0.01	-0.01	-0.07	-0.08
		Loc	0.471	---	0.324	0.324	0.471	0.397

```
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     4      Down    Def       5.34         ---         3.92        3.92        5.34        9.26
                    Loc      5.025         ---        4.875       4.875       5.025       4.950
              Up    Def        ---         ---          ---         ---         ---         ---
                    Loc        ---         ---          ---         ---         ---         ---
     5      Down    Def        ---         ---          ---         ---         ---         ---
                    Loc        ---         ---          ---         ---         ---         ---
              Up    Def      -0.46         ---        -0.25       -0.25       -0.46       -0.71
                    Loc      0.250         ---        0.250       0.250       0.250       0.250

Long-term Deflections
=====================
   Long-term Column Strip Deflection Factors
   -----------------------------------------
   Time dependant factor for sustained loads = 2.000
   Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)
                                  M+ve                                        M-ve
   Span    Zone    Astop          b          d   Rho' Lambda     Asbot          b          d   Rho' Lambda
   ---- ------- -------- -------- -------- ------ ------ -------- -------- -------- ------ ------
      1 Right       ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      2 Midspan     ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      3 Midspan     ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      4 Midspan     ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      5 Left        ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000

   NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
          Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

   Long-term Middle Strip Deflection Factors
   -----------------------------------------
   Time dependant factor for sustained loads = 2.000
   Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)
                                  M+ve                                        M-ve
   Span    Zone    Astop          b          d   Rho' Lambda     Asbot          b          d   Rho' Lambda
   ---- ------- -------- -------- -------- ------ ------ -------- -------- -------- ------ ------
      1 Right       ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      2 Midspan     ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      3 Midspan     ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      4 Midspan     ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000
      5 Left        ----      ----      ----  0.000  2.000     ----      ----      ----  0.000  2.000

   NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
          Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

   Extreme Long-term Column Strip Deflections and Corresponding Locations
   ----------------------------------------------------------------------
   Units: D (mm), x (m)
   Span Direction Value          cs       cs+lu        cs+l       Total
   ---- --------- ----- ----------- ----------- ----------- -----------
      1      Down   Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
               Up   Def       -0.92       -1.17       -1.17       -1.62
                    Loc       0.000       0.000       0.000       0.000
      2      Down   Def       22.15       31.53       31.53       42.61
                    Loc       4.275       4.275       4.275       4.275
               Up   Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
      3      Down   Def        8.31       12.19       12.19       16.34
                    Loc       4.500       4.500       4.500       4.500
               Up   Def       -0.08       -0.08       -0.08       -0.12
                    Loc       0.250       0.250       0.250       0.250
      4      Down   Def       22.15       31.53       31.53       42.61
                    Loc       4.725       4.725       4.725       4.725
               Up   Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
      5      Down   Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
               Up   Def       -0.92       -1.17       -1.17       -1.62
                    Loc       0.250       0.250       0.250       0.250

   NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
          Incremental deflections after partitions are installed can be estimated by deflections due to:
          - creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
          - creep and shrinkage plus live load (cs+l), if live load applied after partitions.
          Total deflections consist of dead, live, and creep and shrinkage deflections.

   Extreme Long-term Middle Strip Deflections and Corresponding Locations
   ----------------------------------------------------------------------
   Units: D (mm), x (m)
   Span Direction Value          cs       cs+lu        cs+l       Total
   ---- --------- ----- ----------- ----------- ----------- -----------
      1      Down   Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
               Up   Def       -0.92       -1.17       -1.17       -1.62
                    Loc       0.000       0.000       0.000       0.000
      2      Down   Def       10.68       14.59       14.59       19.93
                    Loc       3.975       4.050       4.050       3.975
               Up   Def         ---         ---         ---         ---
```

```
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                    Loc         ---         ---         ---         ---
     3      Down    Def        2.63        4.10        4.10        5.41
                    Loc       4.500       4.500       4.500       4.500
              Up    Def       -0.15       -0.15       -0.15       -0.22
                    Loc       0.471       0.397       0.397       0.471
     4      Down    Def       10.68       14.59       14.59       19.93
                    Loc       5.025       4.950       4.950       5.025
              Up    Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
     5      Down    Def         ---         ---         ---         ---
                    Loc         ---         ---         ---         ---
              Up    Def       -0.92       -1.17       -1.17       -1.62
                    Loc       0.250       0.250       0.250       0.250

   NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
          Incremental deflections after partitions are installed can be estimated by deflections due to:
          - creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
          - creep and shrinkage plus live load (cs+l), if live load applied after partitions.
          Total deflections consist of dead, live, and creep and shrinkage deflections.
```

7. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (*kN.m*)			
		Hand (EFM)	**spSlab**
Exterior Span			
Column Strip	Exterior Negative*	310.1	312.3
	Positive	287.6	304.3
	Interior Negative*	773.7	771.7
Middle Strip	Exterior Negative*	0.0	0.0
	Positive	191.7	202.9
	Interior Negative*	164.1	163.7
Interior Span			
Column Strip	Interior Negative*	695.1	693.4
	Positive	156.4	161.1
Middle Strip	Interior Negative*	147.4	147.1
	Positive	104.3	107.4
* negative moments are taken at the faces of supports			

Table 10 - Comparison of Reinforcement Results							
Span Location		**Reinforcement Provided for Flexure**		**Additional Reinforcement Provided for Unbalanced Moment Transfer***		**Total Reinforcement Provided**	
		Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior Span							
Column Strip	Exterior Negative	24-15M	24-15M	n/a	n/a	24-15M	24-15M
	Positive	21-15M	21-15M	n/a	n/a	21-15M	21-15M
	Interior Negative	37-15M	37-15M	---	---	37-15M	37-15M
Middle Strip	Exterior Negative	12-15M	12-15M	n/a	n/a	12-15M	12-15M
	Positive	14-15M	14-15M	n/a	n/a	14-15M	14-15M
	Interior Negative	12-15M	12-15M	n/a	n/a	12-15M	12-15M
Interior Span							
Column Strip	Positive	12-15M	12-15M	n/a	n/a	12-15M	12-15M
Middle Strip	Positive	12-15M	12-15M	n/a	n/a	12-15M	12-15M

Table 11 - Comparison of One-Way (Beam Action) Shear Check Results								
Span	**V_f @ d_v, kN**		**V_f @ drop panel, kN**		**V_c @ d_v , kN**		**V_c @ drop panel, kN**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	576.6	577.2	449.4	439.2	1,506.4	1,587.2	1,465.2	1,465.2
Interior	502.8	503.9	375.6	365.8	1,506.4	1,587.2	1,465.2	1,465.2

* x_u calculated from the centerline of the left column for each span

Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)										
Support	**b_1, mm**		**b_2, mm**		**b_o, mm**		**V_f, kN**		**c_{AB}, mm**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	666	832	832	832	2,164	2,164	465.6	515.5	205.0	205.0
Interior	832	832	832	832	3,328	3,328	1,159.8	1,190.6	416.0	416.0
Corner	666	666	666	666	1,332	1,332	253.9	272.9	166.5	166.5
Support	**J_c, mm^4**		**γ_v**		**M_{unb}, kN.m**		**v_f, MPa**		**v_c, MPa**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	3.93×10^{10}	3.93×10^{10}	0.374	0.374	330.3	317.8	1.29	1.34	1.426	1.426
Interior	1.33×10^{11}	1.33×10^{11}	0.400	0.400	113.8	113.2	1.19	1.22	1.426	1.426
Corner	2.25×10^{10}	2.25×10^{10}	0.400	0.400	184.3	177.4	1.12	1.14	1.426	1.426

Table 13 - Comparison of Two-Way (Punching) Shear Check Results (around Drop Panels)										
Support	**b_1, mm**		**b_2, mm**		**b_o, mm**		**V_f, kN**		**c_{AB}, mm**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	1,862	1,862	3,224	3,224	6,948	6,948	393.6	441.6	205.0	205.0
Interior	3,224	3,224	3,224	3,224	12,896	12,896	1,030.6	1059.2	416.0	416.0
Corner	1,862	1,862	1,862	1,862	3,724	3,724	214.4	231.9	166.5	166.5
Support	**J_c, mm^4**		**γ_v**		**M_{unb}, kN.m**		**v_f, MPa**		**v_c, MPa**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	5.8×10^{11}	5.8×10^{11}	N.A.	N.A.	N.A.	N.A.	0.25	0.28	1.1	1.1
Interior	5.01×10^{12}	5.01×10^{12}	N.A.	N.A.	N.A.	N.A.	0.36	0.37	1.0	1.0
Corner	3.03×10^{11}	3.03×10^{11}	N.A.	N.A.	N.A.	N.A.	0.24	0.28	1.19	1.19

Note: Shear stresses from spSlab are higher than hand calculations since it considers the load effects beyond the column centerline known in the model as right/left cantilevers. This small increase is often neglected in simplified hand calculations like the one used here.

Table 14 - Comparison of Immediate Deflection Results (mm)								
Column Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	12.07	11.07	12.07	11.07	23.87	20.46	11.80	9.39
Interior	3.33	4.15	3.33	4.15	6.34	8.03	3.01	3.88
Middle Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	6.63	5.34	6.63	5.34	13.07	9.26	6.44	3.92
Interior	0.78	1.32	0.78	1.32	1.48	2.78	0.70	1.47

Table 15 - Comparison of Time-Dependent Deflection Results						
Column Strip						
Span	**λ_{Δ}**		**Δ_{cs}, mm**		**Δ_{total}, mm**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	24.14	22.15	48.01	42.61
Interior	2.0	2.0	6.66	8.31	13.00	16.34
Middle Strip						
Span	**λ_{Δ}**		**Δ_{cs}, mm**		**Δ_{total}, mm**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	13.26	10.68	26.33	19.93
Interior	2.0	2.0	1.48	2.63	3.04	5.41

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model.

8. Conclusions & Observations

8.1. One-Way Shear Distribution to Slab Strips

In one-way shear checks above, shear is distributed uniformly along the width of the design strip (9 m). StructurePoint finds it necessary sometimes to allocate the one-way shears with the same proportion moments are distributed to column and middle strips.

spSlab allows the one-way shear check using two approaches: 1) calculating the one-way shear capacity using the average slab thickness and comparing it with the total factored one-shear load as shown in the hand calculations above; 2) distributing the factored one-way shear forces to the column and middle strips and comparing it with the shear capacity of each strip as illustrated in the following figures. An engineering judgment is needed to decide which approach to be used.

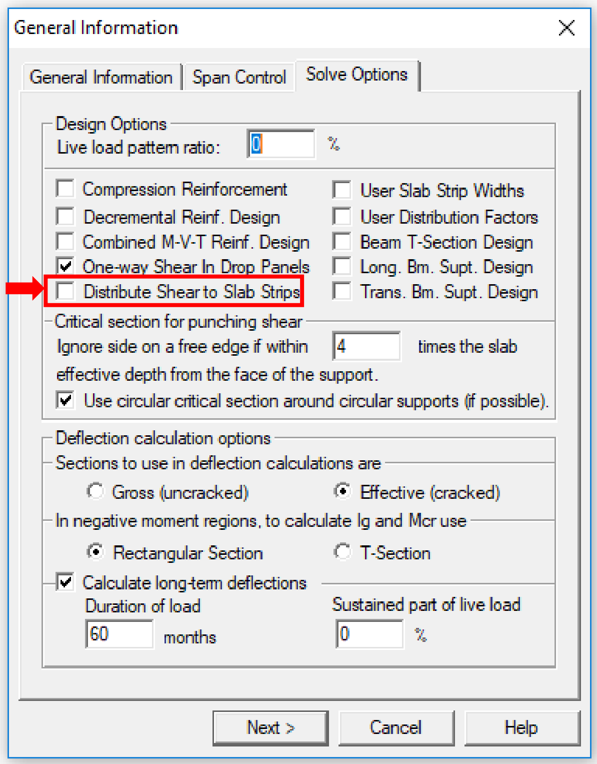
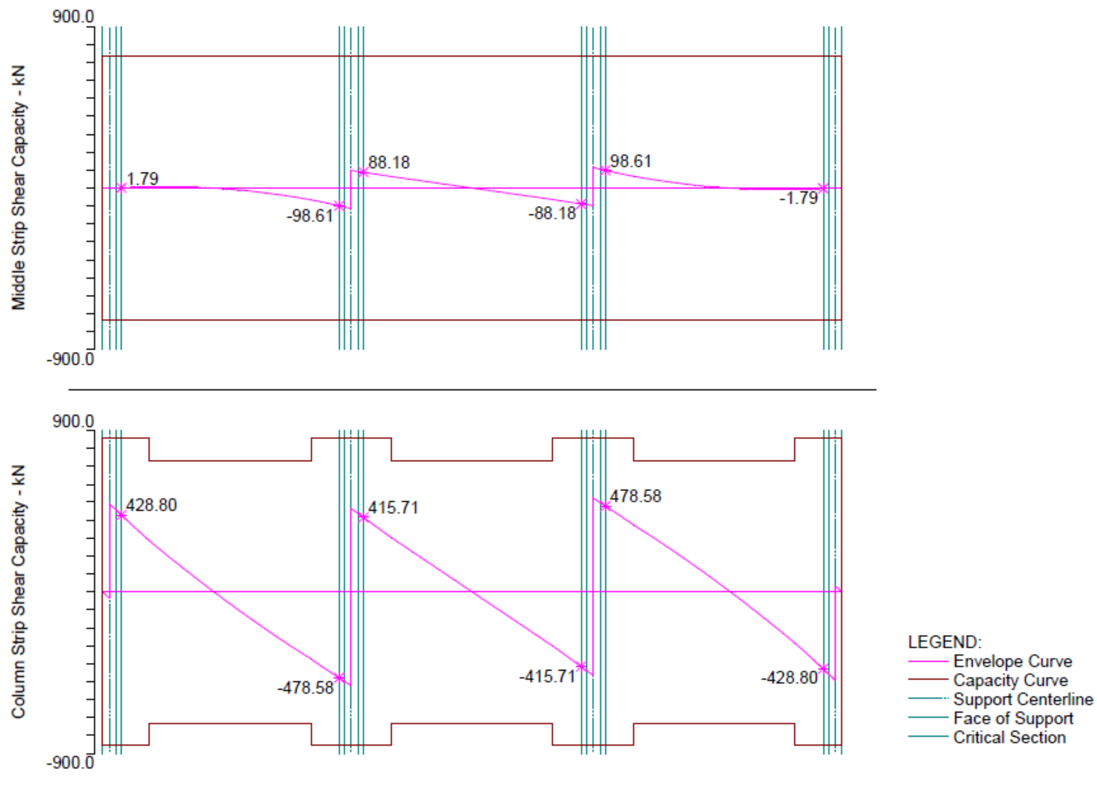


Figure 23a – Distributing Shear to Column and Middle Strips (spSlab Input)



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Project: Two Way Slab with Drop Panels

Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 08/06/18

Time: 16:01:14

Figure 23b – Distributed Column and Middle Strip Shear Force Diagram (spSlab Output)

Slab Shear Capacity
===================

Units: b, dv (mm), Xu (m), PhiVc, Vu(kN)

Span	Strip	b	dv	Beta	Vratio	PhiVc	Vu	Xu
1	Column	4500	202	0.210	1.000	732.60	0.00	0.00
		4500	266	0.210	1.000	854.63	0.00	0.00
	Middle	4500	202	0.210	0.000	732.60	0.00	0.00
		4500	202	0.210	0.000	732.60	0.00	0.00
2	Column	4500	266	0.210	0.996	854.63	428.80	0.45
		4500	202	0.210	0.851	732.60	373.60	7.50
		4500	266	0.210	0.829	854.63	478.58	8.55
	Middle	4500	202	0.210	0.026	732.60	7.53	1.50
		4500	202	0.210	0.149	732.60	65.55	7.50
		4500	202	0.210	0.171	732.60	98.61	8.55
3	Column	4500	266	0.210	0.825	854.63	415.71	0.45
		4500	202	0.210	0.825	732.60	301.83	7.50
		4500	266	0.210	0.825	854.63	415.71	8.55
	Middle	4500	202	0.210	0.175	732.60	88.18	0.45
		4500	202	0.210	0.175	732.60	64.02	7.50
		4500	202	0.210	0.175	732.60	88.18	8.55
4	Column	4500	266	0.210	0.829	854.63	478.58	0.45
		4500	202	0.210	0.851	732.60	373.60	1.50
		4500	266	0.210	0.996	854.63	428.80	8.55
	Middle	4500	202	0.210	0.171	732.60	98.61	0.45
		4500	202	0.210	0.149	732.60	65.55	1.50
		4500	202	0.210	0.026	732.60	7.53	7.50
5	Column	4500	266	0.210	1.000	854.63	0.00	0.25
		4500	202	0.210	1.000	732.60	0.00	0.25
	Middle	4500	202	0.210	0.000	732.60	0.00	0.25
		4500	202	0.210	0.000	732.60	0.00	0.25

Figure 23c – Tabulated Shear Force & Capacity at Critical Sections (spSlab Output)

8.2. Two-Way Concrete Slab Analysis Methods

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in CSA A.23.3-14 Clause 13.

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of ***CSA A.23.3-14 (13.9.1)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Elastic Frame Method (EFM) does not have the limitations of DDM. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the EFM to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable CSA A23.3-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
13.8.1.1 13.9.1.1	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
13.8.1.1 13.9.1.1	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑	☑	
13.8.1.1 13.9.1.1	Column offset shall not exceed 20% of the span in direction of offset from either axis between centerlines of successive columns	☑	☑	
13.8.1.1 13.9.1.1	The reinforcement is placed in an orthogonal grid.	☑	☑	
13.9.1.2	Minimum of three continuous spans in each direction	☑		
13.9.1.3	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
13.9.1.4	All loads shall be due to gravity only	☑		
13.9.1.4	All loads shall be uniformly distributed over an entire panel (q_f)	☑		
13.9.1.4	Factored live load shall not exceed two times the factored dead load	☑		
13.10.6	Structural integrity steel detailing	☑	☑	☑
13.10.10	Openings in slab systems	☑	☑	☑
8.2	Concentrated loads	Not permitted	☑	☑
13.8.4.1	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
13.10.2*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
13.8.2	Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)	Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).